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BUILDING HEAT TRANSFER

Understanding Through Numerical Examples

BEEP
BUILDING ENERGY
EFFICIENCY PROJECT

UDAYRAJ

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Building Heat Transfer:
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Disclaimer:
This publication has been developed after an extensive review of all relevant data and documents and in consultation with a number of experts and stakeholders of the building energy sector, both in India and Switzerland. The analysis, interpretations, and recommendations expressed herein do not necessarily reflect the view of the Bureau of Energy Efficiency (BEE) and the Swiss Agency for Development and Cooperation (SDC). BEE and SDC disclaim liability for any personal injury, property, or other damages of any nature whatsoever, whether special, indirect, consequential, or compensatory, directly or indirectly resulting from the publication, use of, application, or reliance on this document.

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TABLE OF CONTENTS

MESSAGES

Bureau of Energy Efficiency
(Ministry of Power, Government of India)

MESSAGE

Abhay Bakre
Director General, Bureau of Energy Efficiency

For a country like India, the developmental prospects are inextricably linked to energy. As we know, rapid urbanization and growth in the building infrastructure will lead to an increase in energy demand in buildings by many folds. Currently, buildings account for 35% of the total energy consumption in the country and is growing at 8% annually. As per the projections made by NITI Aayog, the electricity consumption in the residential sector is expected to increase by 6–13 times, and the same for the commercial sector by 7–11 from 2012 to 2047, under different scenarios. This shows that there is a huge opportunity of energy savings in buildings.

Bureau of Energy Efficiency is focusing on the building sector and has adopted a multi-pronged approach to reduce the energy consumption in the building sector. This includes regulation for commercial buildings (Energy Conservation Building Code or ECBC), as well as for residential buildings (Eco-Niwas Samhita or ENS), Standards and Labelling Programme for Appliances and Buildings, Energy Auditor (Buildings) programme and various awareness generation and capacity building initiatives.

In addition to the above initiatives, BEE is also keen to develop the next generation of building sector professionals who can help in achieving BEE's energy savings targets. The book – *Building Heat Transfer: Understanding through Numerical Examples* – is an important step towards this initiative. This book is a compilation of 20 unique practical problems, which are developed using the data from real-life case studies of buildings and I am confident that it will help in creating interest among undergraduate / post-graduate engineering students in the building sector and guide them to link the technical knowledge with the real-life application in buildings.

I hope this book gets widely adopted by various engineering institutes and becomes a key reference document for the faculty as well as the students who are interested in heat transfer in buildings.

New Delhi
4th December 2021

Abhay Bakre
Director General
Bureau of Energy Efficiency

Schweizerische Eidgenossenschaft
Confédération suisse
Confederazione Svizzera
Confederaziun svizra

Swiss Agency for Development
and Cooperation SDC

MESSAGE

Jonathan Demenge
Head of Cooperation and Counsellor

1. The purpose of this publication is simple: equip engineers and architects with a fundamental grasp of energy efficiency principles by anchoring these principles into the curricula of Indian engineering and architecture institutions.
2. The importance of this publication derives from a stark fact: the buildings and construction sector are responsible for more than one-third of global energy use and 40% of carbon dioxide (CO_2) emissions. Moreover, a sharp increase in energy consumption in buildings is being observed, particularly in countries with fast growing economies such as India. With average temperatures rising globally because of climate change, buildings' energy consumption and CO_2 emissions are bound to increase, while buildings may fail to provide adequate thermal comfort. Therefore, it is imperative to integrate energy efficiency measures into buildings.
3. This publication – *Building Heat Transfer: Understanding through Numerical Examples* – builds on the work carried out under the Indo-Swiss Building Energy Efficiency Project (BEEP), which is implemented under a collaboration between the Bureau of Energy Efficiency (BEE) and the Swiss Agency for Development and Cooperation (SDC).
4. For four decades, Switzerland has been at the forefront of promoting energy efficiency in the building sector: by adopting ambitious energy efficiency measures, Switzerland was able to reduce energy consumption in new buildings by 75%. SDC with Swiss and Indian partners endeavour to bring in this experience, adapt it, co-develop knowledge and expertise, and translate these into energy efficient building design, technologies, and policies in India through the BEEP project.
5. To promote energy efficiency and have impacts, the knowledge and expertise must be fed into existing curricula and fill in existing gaps in academic programmes. As the buildings sector need future engineers and architects, BEEP focuses – among other things – on the academic sector to expose students to the challenging problems of building science.
6. I would like to express my admiration and my gratitude to the BEEP team for their innovative and dedicated work. I hope that engineers, architects, students, and faculties of the Indian institutions will find this publication useful in their work, in their teaching, and in shaping the future of the buildings sectors and energy consumption.

New Delhi
4th December 2021

Jonathan Demenge, PhD
Head of Cooperation and Counsellor
Swiss Cooperation Office India
Embassy of Switzerland, New Delhi

Bureau of Energy Efficiency
(Ministry of Power, Government of India)

MESSAGE

Saurabh Diddi
Director

I am happy to be a part of the development process of this publication as it is an important addition to the ecosystem of working towards energy efficient buildings. This book introduces undergraduate/postgraduate (UG/PG) engineering students to the building science area. To make concepts clearer to students, 20 problems and their solutions have been presented in this book. These problems are developed with a view that they serve as good examples for the professional core subject, Heat and Mass Transfer, taught in engineering. Students' understanding of the fundamental concepts of this core subject will improve further when they apply themselves to solve these problems. The problems presented include examples from various topics of the subject such as conduction, convection, and radiation modes of heat transfer while touching some of the basic aspects related to thermal comfort and passive building design strategies. The integration of Building Heat Transfer Book in the academic curricula will enable the students to have an applied understanding of building physics.

I take this opportunity the thank the main author of this book, Dr Udayraj, Assistant Professor, IIT Bhilai, who worked tirelessly to compile these problem sets keeping a link to the real-life building examples. I would also like to thank Prof. Uday N Gaitonde (Professor, IIT Bombay) and Dr Dibakar Rakshit (Associate Professor, IIT Delhi) for thoroughly reviewing the drafts of this publication and providing their inputs, which were duly incorporated by the author.

I would also like to thank the Swiss Agency for Development and Cooperation (SDC) and Indo-Swiss collaboration on Building Energy Efficiency Project (BEEP) for supporting the Bureau of Energy Efficiency (BEE) in developing this book. The BEEP team comprised Dr Sameer Maithel, Mr Prashant Bhanware, and Mr Mohit Jain, who worked closely with Dr Udayraj for the development of this book.

Lastly, I would like to thank Mr Abhay Bakre, DG, BEE, for his kind guidance and my colleague, Ms Akanksha Krishan, for closely coordinating with the development team.

New Delhi
4th December 2021

Saurabh Diddi
Director
Bureau of Energy Efficiency

A NOTE FROM THE AUTHOR

The Indo-Swiss Building Energy Efficiency Project (BEEP) is a bilateral cooperation project between the Federal Department of Foreign Affairs (FDFA) of the Swiss Confederation and the Ministry of Power (MoP), Government of India. The overall goal of the project is to reduce energy consumption in new commercial, public, and residential buildings in India through energy-efficient and thermally comfortable (EETC) design. The mandates of BEEP are to (a) design EETC buildings, (b) work on building technologies and policies, and (c) effectively transfer the knowledge to targeted stakeholder groups. This publication is a part of the broader efforts of BEEP to improve building science education in India and to develop skilled manpower in the domain of building energy efficiency.

Energy-efficient buildings are going to play a significant role towards realising the sustainable development goals. To design energy-efficient buildings, it is important that both architects and engineers work together from the early phase of designing a building. In this regard, it is important for design engineers to have basic understanding of the building science. Unfortunately, the interest and inclination of most of the talented engineers in such an interesting problem is somehow lacking. This may be partially due to lack of exposure at the undergraduate (UG) and postgraduate (PG) levels of education where most of the courses are still taught with a major focus on conventional/industrial engineering problems. So, it is essential to expose them now to the challenging problems related to building science while aligning it with course contents of their professional core courses.

To inculcate a basic understanding and to introduce UG/PG engineering students with the building science area, 20 problems have been developed and presented in this book. These problems are developed with a view that they serve as good examples for the professional core subject, Heat and Mass Transfer, taught in engineering. I believe that students' understanding of the fundamental concepts of this core subject will improve further when they apply themselves to solve these problems. The problems presented include examples from various topics of the subject such as conduction, convection, and radiation modes of heat transfer while touching some of the basic aspects related to thermal comfort and passive building design strategies. Necessary care has been taken, and appropriate assumptions have been made at various places in a few problems to simplify them, so that UG/PG students can understand and appreciate them while learning the core ideas of building physics. The sets of solution presented in the book would make concepts clearer to practising architects and engineers.

Special care is taken to keep the problems as practical as possible and India-centric. Problems and solutions were developed based on the data from real-life case studies of buildings taken from the BEEP project, at different site locations in India. Further, the input data for some of the problems are taken from real experimentation and building simulations. Remarks provided at the end of each solution will be helpful for the students to learn new concepts and to relate the subject with their practical observations as far as buildings are concerned. I sincerely hope that students will find this book interesting and educators will find it meaningful and useful. As mentioned earlier, the problems are developed keeping the course content in mind and contains problems related to various key concept of the subject. To help the students and educators further, all the developed problems are mapped with various topics of the Heat and Mass Transfer course in the following table. I cordially welcome your suggestions and comments for further improvements. Please feel free to write at udayraj@iitbhilai.ac.in

I would like to thank Greentech Knowledge Solutions Pvt. Ltd (GKSPL) and BEEP for providing this opportunity and for supporting the work. Regular discussions, expert suggestions, and critical comments from Dr Sameer Maithel and Mr Prashant Bhanware have helped immensely in framing the problems. Mr Prashant Bhanware has provided some critical input parameters for the problems. Mr Shubham Giri has supported me throughout the solution development and revision process. Thanks are due to Prof. Uday N Gaitonde (Professor, IIT Bombay) and Dr Dibakar Rakshit (Associate Professor, IIT Delhi) for agreeing to peer-review the problems. Their excellent suggestions and comments have helped in improving the quality of the final output. Finally, I would like to extend my sincere thanks to my family, Director, HoD and colleagues of Mechanical Engineering Department for their positive support and encouragement.

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MAPPING OF THE DEVELOPED PROBLEM WITH HEAT AND MASS TRANSFER COURSE

Relevant problem in the booklet (Problem no.)	Relevant course content (Topic) of Heat and Mass Transfer subject	Basic idea related to building physics to be conveyed through the solution
Problem 1 and Problem 2	Conduction heat transfer – Multilayer	Effect and importance of building wall insulation on U-value/heat transfer in the absence and presence of solar radiation.
Problem 3	Conduction heat transfer – Multilayer	Effect and importance of providing insulation in roof on heat transfer and inner surface temperature.
Problem 4	Conduction heat transfer – Multilayer	How different walls and roof contribute to building cooling load with diurnal variation? Which wall of the building is more crucial from the cooling load point of view?
Problem 5	Conduction heat transfer – Multilayer	Effectiveness of three different roof constructions (business as usual, reflective tiles, and painted roofs) during summer in hot climate of India.
Problem 6	Conduction heat transfer – Multilayer	Effect of roof surface absorptivity on heat transfer.
Problem 7	Convection heat transfer	Effect of airtightness of the building structure on cooling load.
Problem 8	Forced convection + Conduction heat transfer – Multilayer	Effect of wind speed on heat transfer through insulated and uninsulated building walls.
Problem 9	Forced convection + Conduction heat transfer – Multilayer + Heat transfer rate equations	Quantifying the error in the measurement of room air temperature due to radiation interference. Introduction to the concept of radiation shield in temperature measurement.
Problem 10	Radiation heat transfer	Effect of building orientation on incident solar radiation and cooling load.
Problem 11	Radiation heat transfer	Dominance of radiative heat transfer through a single-layer glass window and its role in building cooling load. Introduction to the concept of solar heat gain coefficient (SHGC).

Relevant problem in the booklet (Problem no.)	Relevant course content (Topic) of Heat and Mass Transfer subject	Basic idea related to building physics to be conveyed through the solution
Problem 12	Radiation heat transfer + Convective heat transfer	Heat transfer through a double-pane glass window and its comparison with single-pane glass window. How effective is the double-pane window compared to a single-pane window?
Problem 13	Radiation heat transfer + Convective heat transfer	Introducing students to the concept and importance of sol-air temperature. How sol-air temperature is affected by wall location and time of the day?
Problem 14	Radiation heat transfer + Conduction heat transfer – Multilayer	Impact of window-to-wall ratio on overall heat transfer through a building envelope. Developing a thought process for an optimum window size considering cooling load, natural ventilation, and daylighting.
Problem 15	Multimode heat transfer (Conduction, convection, and radiation heat transfer)	Introducing students to the concept and importance of external shading in windows. Identification of the best method for reducing heat transfer through window between an external shading system and a double-pane glass window.
Problem 16	Radiation heat transfer – Shape factor	Introducing students to the concept of mean radiant temperature (MRT). Effect of shape factor on MRT. Effect of positioning of an occupant in a living space on his/her thermal comfort.
Problem 17	Radiation heat transfer – Shape factor Radiation heat transfer – Radiation exchange between surfaces	Introducing students to the concept of radiant cooling system used in buildings. Demonstrating effectiveness of radiant cooling technique to achieve human thermal comfort in buildings.
Problem 18	Radiation heat transfer – Radiation exchange between surfaces	Comparison of a conventional air conditioning system with a radiant cooling system based on human thermal comfort and building energy efficiency. Introducing students to the importance of set-point temperature in conventional air-conditioning systems and its effect of building energy consumption.

Relevant problem in the booklet (Problem no.)	Relevant course content (Topic) of Heat and Mass Transfer subject	Basic idea related to building physics to be conveyed through the solution
Problem 19	Radiation heat transfer – Radiation exchange between surfaces	Comparison of human thermal comfort at the ground floor and the top floor of a two-story building. Why we feel hotter or thermally uncomfortable sometimes in the upper floor compared to the ground floor?
Problem 20	Multimode heat transfer (Conduction, convection, radiation, and evaporation heat transfer)	Introduction to heat balance of the human body and its role in achieving thermal comfort. Determination of human skin and core body temperatures using simple calculations. Effect of building wall insulation on human thermal comfort.

BUILDING HEAT TRANSFER:
20 PROBLEMS AND SOLUTIONS

PROBLEM 1

In this era of energy crisis, like developed countries, India is also looking to build energy-efficient buildings. One such building in India is Aranya Bhawan, at Jaipur. Consider a typical room in the building,which has only one wall exposed to the ambient (i.e., external wall). The external wall consists of a layer of XPS insulation of 50 mm between two brick layers of 115 mm each. The brick wall is plastered with 15-mm-thick cement plaster on inside and outside surfaces. The thermal conductivities of insulation, brick layer, and plaster layer are 0.028 W/mK, 0.85 W/mK, and 0.72 W/mK, respectively. The average heat transfer coefficients of inner and outer walls are 7.69 W/m^{2}K and 25.0 W/m^{2}K, respectively. The room is maintained at 27 °C to achieve thermal comfort and the dimension of its external wall is 5.0 × 3.5 m. Considering average environmental conditions at 9.00 p.m. during the summer month of May in Jaipur (ambient temperature 34 °C and no solar radiation), answer the following:

- Determine the overall heat transfer coefficient, the heat transfer rate through the wall, and temperatures of the inner and outer surfaces of the wall assembly considering steady state condition.
- Repeat the calculation for a normal construction with 230-mm-thick external brick wall, with 15-mm plaster on either side without any insulation. Compare and discuss the obtained results with an insulated external wall case.

SOLUTION

Given:

Thermal conductivity and thickness of plaster, k_p = 0.72 W/mK, L_p = 0.015 m

Thermal conductivity and thickness of brick, k_b = 0.85 W/mK, L_b = 0.115 m

Thermal conductivity and thickness of XPS insulation, k_{XPS} = 0.028 W/mK, L_{XPS} = 0.05 m

Heat transfer coefficient of inner walls, h_i = 7.69 W/m^{2}K

Heat transfer coefficient of outer walls, h_o = 25 W/m^{2}K

Indoor room temperature, $T_{\infty, i}$ = 27 °C

Ambient temperature, $T_{\infty, o}$ = 34 °C

Radiation heat transfers between the outer wall and the surroundings, and the inner wall and room walls are neglected.

(i) With Insulation Case:

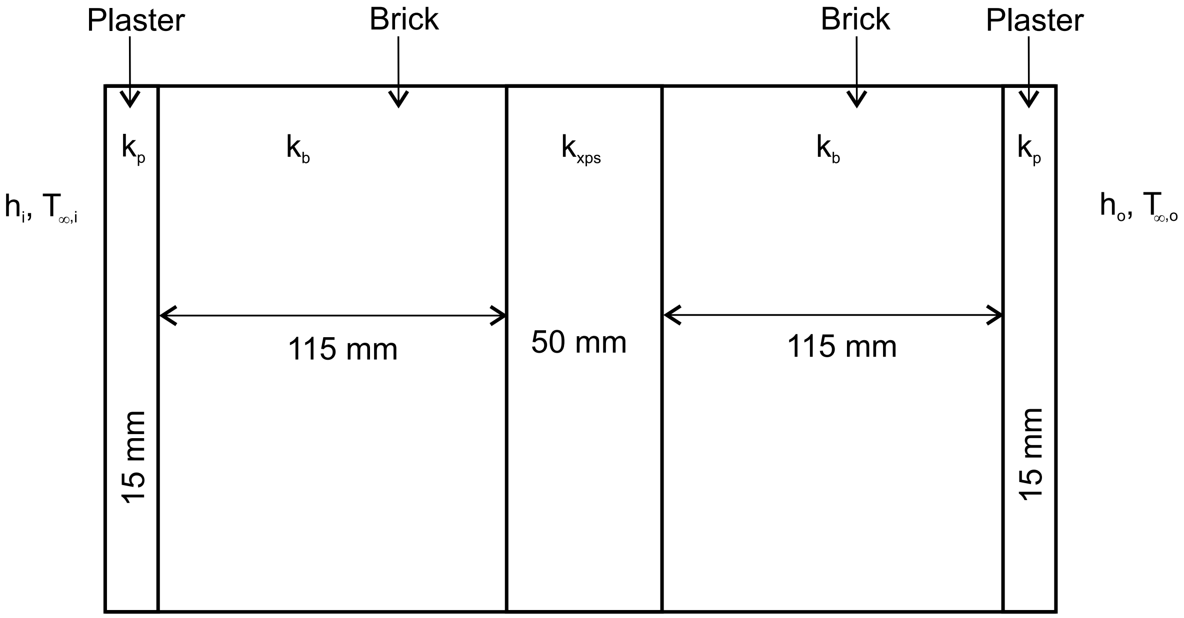

Figure 1.1

Total thermal resistance,

$$R_{total} = \frac{1}{h_i A} + \frac{L_p}{k_p A} + \frac{L_b}{k_b A} + \frac{L_{XPS}}{k_{XPS} A} + \frac{L_b}{k_b A} + \frac{L_p}{k_p A} + \frac{1}{h_o A} \quad \text{...(1.1)}$$

$$R_{total} = \frac{1}{7.69A} + \frac{0.015}{0.72A} + \frac{0.115}{0.85A} + \frac{0.05}{0.028A} + \frac{0.115}{0.85A} + \frac{0.015}{0.72A} + \frac{1}{25A}$$

$$R_{total} = \frac{2.27}{A} K/W$$

Overall heat transfer coefficient,

$$U_{overall} = \frac{1}{R_{total} A} = 0.44 \frac{W}{m^2 K} \quad \text{...(1.2)}$$

Heat transfer rate,

$$q = \frac{\Delta T}{R_{total}} = \frac{34-27}{2.27} A = 3.08 \times (5 \times 3.5) = 53.90\ W \quad \text{...(1.3)}$$

Heat flux,

$$q" = \frac{q}{A} = \frac{53.90}{5 \times 3.5} = 3.08\ W/m^2 \quad \text{...(1.4)}$$

Temperature of the inner surfaces of the wall assembly, $T_{in,wall}$

$$q" = h_i \left(T_{in,wall} - T_{\infty,i} \right) \quad \text{...(1.5)}$$

$$T_{in,wall} = \frac{3.08}{7.69} + 27 = 27.04\ °C \quad \text{...(1.6)}$$

Temperature of the outer surfaces of the wall assembly, $T_{out,wall}$

$$q'' = h_o\left(T_{\infty,o} - T_{out,wall}\right) \qquad \textbf{...(1.7)}$$

$$T_{out,wall} = 34 - \frac{3.08}{25} = 33.88\ ^\circ\mathrm{C} \qquad \textbf{...(1.8)}$$

(ii) Without Insulation Case:

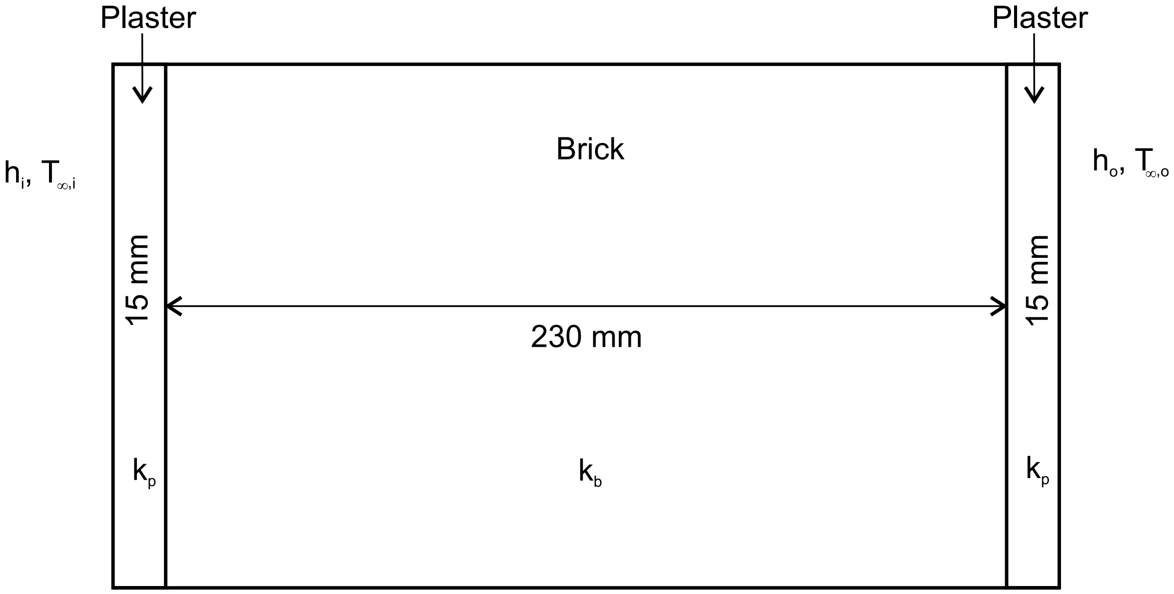

Figure 1.2

Total thermal resistance,

$$R_{total} = \frac{1}{h_i A} + \frac{L_p}{k_p A} + \frac{L_b}{k_b A} + \frac{L_p}{k_p A} + \frac{1}{h_o A} \qquad \textbf{...(1.9)}$$

$$R_{total} = \frac{1}{7.69A} + \frac{0.015}{0.72A} + \frac{0.23}{0.85A} + \frac{0.015}{0.72A} + \frac{1}{25A}$$

$$R_{total} = \frac{0.48}{A}\mathrm{K / W}$$

Overall heat transfer coefficient,

$$U_{overall} = \frac{1}{R_{total}A} = 2.08\frac{W}{m^2K} \qquad \textbf{...(1.10)}$$

Heat transfer rate,

$$q = \frac{\Delta T}{R_{total}} = \frac{34-27}{0.48}A = 14.58\times\left(5\times3.5\right) = 255.15\ W \qquad \textbf{...(1.11)}$$

Heat flux,

$$q'' = \frac{q}{A} = \frac{53.90}{5\times3.5} = 14.58\ W / m^2 \qquad \textbf{...(1.12)}$$

Temperature of the inner surfaces of the wall assembly, $T_{in,wall}$

$$q'' = h_i\left(T_{in,wall} - T_{\infty,i}\right) \qquad \textbf{...(1.13)}$$

$$T_{in,wall} = \frac{14.58}{7.69} + 27 = 28.90\ °\text{C}$$

Temperature of the outer surfaces of the wall assembly, $T_{out,wall}$

$$q'' = h_o\left(T_{\infty,o} - T_{out,wall}\right) \qquad \textbf{...(1.14)}$$

$$T_{out,wall} = 34 - \frac{14.58}{25} = 33.42\ °\text{C}$$

REMARKS

It can be observed that the overall heat transfer coefficient (U-value) associated with the wall assembly and the heat transfer through the wall decreased significantly with the introduction of insulation layer in the wall assembly. It can be observed that the thermal resistance offered by XPS insulation (1.785/A K/W) contributes significantly to the total thermal resistance (2.268/A K/W) of the wall incorporating insulation. This shows that the cooling load of the building can be reduced significantly by incorporating insulation in the outer-wall assemblies. The U-values without and with insulation are found to be 2.08 W/m^{2}K and 0.44 W/m^{2}K, respectively. The U-value with insulation is equal to the U-value suggested by the ECBC (0.44 W/m^{2}K). The inner wall surface temperature decreased from 28.9 °C to 27.4 °C with the introduction of insulation in the external wall.

PROBLEM 2

To maintain the comfort environment in a room that has one external wall (west wall exposed to the ambient), a layer of XPS insulation of 50 mm is given between two brick layers of 115 mm each. The brick wall is plastered with 15-mm-thick cement plaster on inside and outside surfaces. The thermal conductivities of insulation, brick layer, and plaster layer are 0.028 W/mK, 0.85 W/mK, and 0.72 W/mK, respectively. The average heat transfer coefficients of inner and outer walls are 7.69 W/m^{2}K and 25.0 W/m^{2}K, respectively. The room with exposed west wall of 5.0 × 3.5 m is maintained at 27 °C to achieve thermal comfort. On a typical day in May at 4.00 p.m. in Jaipur, the incident solar radiation on the west wall is 680 W/m^2 and ambient temperature is 39 ºC. The absorptivity of the exposed surface of the plaster is 0.65.

- Determine the heat transfer rate through the wall and temperatures of the inner and outer surfaces of the wall assembly considering steady state condition.
- Repeat the calculation for a normal construction with 230-mm-thick external brick wall, with 15-mm plaster on either side without any insulation. Compare and discuss the obtained results with the insulated external wall case.

SOLUTION

Given:

Thermal conductivity and thickness of plaster, k_p = 0.72 W/mK, L_p = 0.015 m

Thermal conductivity and thickness of brick, k_b = 0.85 W/mK, L_b = 0.115 m

Thermal conductivity and thickness of XPS insulation, k_{XPS} = 0.028 W/mK, L_{XPS} = 0.05 m

Heat transfer coefficient of inner walls, h_i = 7.69 W/m^{2}K

Heat transfer coefficient of outer walls, h_o = 25 W/m^{2}K

Indoor room temperature, $T_{\infty,i}$ = 27 °C

Ambient temperature, $T_{\infty,o}$ = 39 °C

Cross-sectional area of the wall, A = 5 × 3.5 m^2

Incident solar radiation on the wall, q_{solar} = 680 W/m^2

Absorptivity, α = 0.65

Radiation heat transfers between the outer wall and the surroundings, and the inner wall and room walls are neglected.

(i) With Insulation Case:

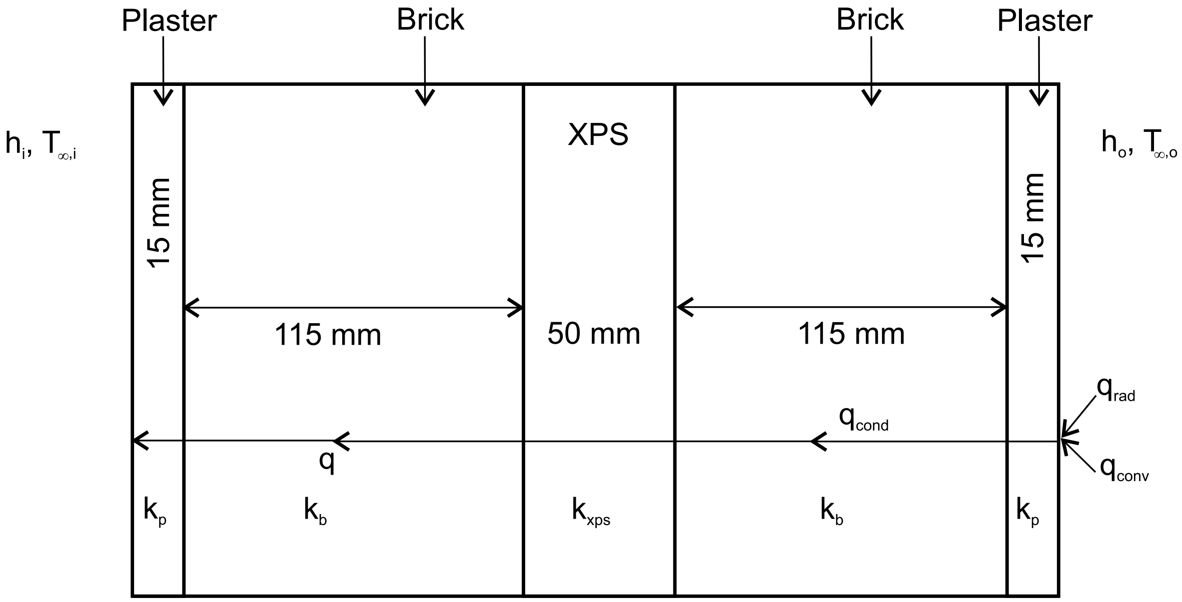

Figure 2.1

Conductive thermal resistance,

$$R_{cond} = \frac{L_p}{k_p A} + \frac{L_b}{k_b A} + \frac{L_{XPS}}{k_{XPS} A} + \frac{L_b}{k_b A} + \frac{L_p}{k_p A} \quad \text{...(2.1)}$$

$$R_{cond} = \frac{0.015}{0.72A} + \frac{0.115}{0.85A} + \frac{0.05}{0.028A} + \frac{0.115}{0.85A} + \frac{0.015}{0.72A}$$

$$R_{cond} = \frac{2.10}{A}\frac{K}{W}$$

Convective thermal resistance:

Inner-wall convective thermal resistance,

$$R_{conv,in} = \frac{1}{h_i A} = \frac{1}{7.69A} = \frac{0.13}{A}\frac{K}{W} \quad \text{...(2.2)}$$

Outer-wall convective thermal resistance,

$$R_{conv,out} = \frac{1}{h_0 A} = \frac{1}{25A} = \frac{0.04}{A}\frac{K}{W} \quad \text{...(2.3)}$$

From the heat balance at the outer surface of the west wall for steady state condition:

$$q_{rad} + q_{conv} = q_{cond} = q \qquad \textbf{...(2.4)}$$

$$680 \times 0.65 \times A + \frac{39 - T_{out,w}}{0.04 \big/ A} = \frac{T_{out,w} - 27}{(2.10 + 0.13) \big/ A}$$

$$442 + 975 - 25\ T_{out,w} = 0.45\ T_{out,w} - 12.11$$

$$T_{out,w} = 56.15\ °\mathrm{C}$$

Heat transfer rate,

$$q = \frac{T_{out,w} - \mathrm{T}_{\infty,\mathrm{i}}}{R_{cond} + R_{conv,in}} = \frac{56.15 - 27}{2.10 + 0.13} A = 13.07 \times \left(5 \times 3.5\right) = 228.725\ W \qquad \textbf{...(2.5)}$$

Heat flux,

$$q'' = \frac{q}{A} = \frac{228.725}{5 \times 3.5} = 13.07\ W / m^2 \qquad \textbf{...(2.6)}$$

Temperature of the inner surfaces of the wall assembly, $\mathrm{T}_{\mathrm{in,w}}$

$$q'' = h_i \left(T_{in,w} - T_{\infty,i} \right) \qquad \textbf{...(2.7)}$$

$$T_{in,w} = \frac{13.07}{7.69} + 27 = 28.70\ °\mathrm{C}$$

(ii) Without Insulation Case:

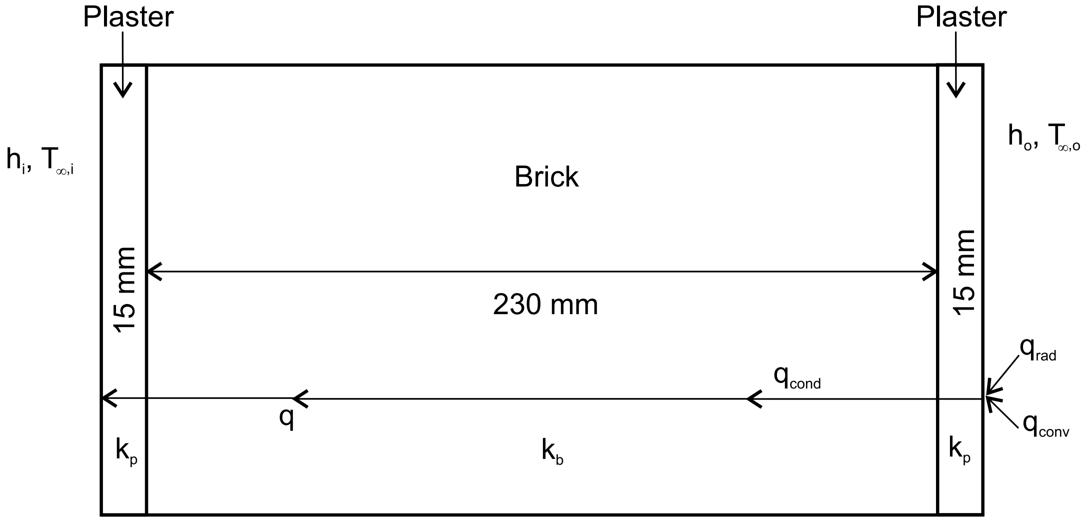

Figure 2.2

Conductive thermal resistance,

$$R_{cond} = \frac{L_p}{k_p A} + \frac{L_b}{k_b A} + \frac{L_p}{k_p A} \quad \textbf{...(2.8)}$$

$$R_{cond} = \frac{0.015}{0.72A} + \frac{0.23}{0.85A} + \frac{0.015}{0.72A}$$

$$R_{cond} = \frac{0.31}{A}\frac{K}{W}$$

Convective thermal resistance:

Inner-wall and outer-wall convective thermal resistances remain the same as with the insulation case.

From the heat balance at the outer surface of the west wall for steady state condition:

$$q_{rad} + q_{conv} = q_{cond} = q \quad \textbf{...(2.9)}$$

$$680 \times 0.65 \times A + \frac{39 - T_{out,w}}{0.04/A} = \frac{T_{out,w} - 27}{(0.31 + 0.13)/A}$$

$$442 + 975 - 25\ T_{out,w} = 2.27\ T_{out,w} - 61.36$$

$$T_{out,w} = 54.21\ °\mathrm{C}$$

Heat transfer rate,

$$q = \frac{T_{out,w} - \mathrm{T}_{\infty,\mathrm{i}}}{R_{cond} + R_{conv,in}} = \frac{54.21 - 27}{0.31 + 0.13} A = 61.84 \times (5 \times 3.5) = 1082.2\ W \quad \textbf{...(2.10)}$$

Heat flux,

$$q" = \frac{q}{A} = \frac{1082.2}{5 \times 3.5} = 61.84\ W/m^2 \quad \textbf{...(2.11)}$$

Temperature of the inner surfaces of the wall assembly, $\mathrm{T}_{\mathrm{in,w}}$

$$q" = h_i \left(T_{in,w} - T_{\infty,i} \right) \quad \textbf{...(2.12)}$$

$$T_{in,w} = \frac{61.84}{7.69} + 27 = 35.04\ °\mathrm{C}$$

REMARKS

It can be observed that the heat transfer through the wall decreased significantly with the introduction of insulation layer in the wall assembly (1082 W to 229 W). This shows that the cooling load of the building can be reduced significantly by incorporating insulation in the outer-wall assemblies. At 4.00 p.m. (with solar radiation), the heat transfer rate decreased from 1082 W to 229 W (78.9%) due to introduction of insulation. Where as it was observed in Question 1 that at 9.00 p.m. (without solar radiation), the heat transfer rate decreased from 255 W to 54 W (78.9%) due to introduction of insulation. This shows that the effect of providing insulation in external wallson heat transfer is significant throughout the day and the percentage change in heat transfer remains similar. The inner-wall surface temperature was found to be significantly lower (28.7 °C) for the insulated wall as compared to the wall without insulation (35.04 °C). Compared to the case without solar radiation (Problem 1), significantly higher heat transfer rate and inner-wall surface temperatures can be noticed when solar radiation is present during the daytime.

PROBLEM 3

Roof of any building is mostly exposed to solar radiation and hence contributes heavily to the building cooling load compared to the external walls of the building. In the present problem, the roof configuration of Aranya Bhawan (Jaipur) and environmental conditions of Jaipur are considered. To decrease the heat transfer through the roof and to minimise the building cooling load, a 40-mm-thick PUF (polyurethane foam) insulation layer is provided on top of the cement screeding of the roof. To reduce absorption of solar radiation, a layer of 15-mm-thick light-coloured terrazzo tile is provided on top of a 50-mm concrete layer. At the bottom of the 20-mm cement screeding layer, there is a layer of 150-mm-thick RCC slab. The bottom-most layer of the roof (inner-most layer) is plaster of 15-mm thickness. The average heat transfer coefficients associated with inner and outer roof sides are 5.88 W/m^{2}K and 25.0 W/m^{2}K, respectively. The room temperature is maintained at 27 °C to achieve thermal comfort. The ambient temperature and solar radiation during summer (May) in Jaipur at 1.00 p.m. are 38.0 °C and 1012 W/m^2, respectively. The absorptivity of the exposed surface of the tile is 0.4 and the exposed surface area of the roof is 5 × 5 m.

- Find out the heat transfer rate to the room through the roof and temperatures of the inner and outer surfaces of the roof assembly considering steady state condition.
- Calculate all the above parameters when the 40-mm-thick PUF insulation layer is not provided between the concrete and the cement screeding. Compare and discuss the results obtained for roof with and without insulation.

Take thermal conductivities of terrazzo tile, concrete, PUF insulation, cement screeding, RCC slab, and plaster as 1.50 W/mK, 1.40 W/mK, 0.023 W/mK, 0.72 W/mK, 1.58 W/mK, and 0.72 W/mK, respectively.

SOLUTION

Given:

Thermal conductivity and thickness of tile, $k_t = 1.5$ W/mK, $L_t = 0.015$ m

Thermal conductivity and thickness of concrete, $k_c = 1.4$ W/mK, $L_c = 0.05$ m

Thermal conductivity and thickness of PUF insulation, $k_{PUF} = 0.023$ W/mK, $L_{PUF} = 0.04$ m

Thermal conductivity and thickness of cement screeding, $k_{cs} = 0.72$ W/mK, $L_{cs} = 0.02$ m

Thermal conductivity and thickness of RCC slab, $k_{RCC} = 1.58$ W/mK, $L_{RCC} = 0.15$ m

Thermal conductivity and thickness of plaster, $k_p = 0.72$ W/mK, $L_p = 0.015$ m

Heat transfer coefficient of inner roof, $h_i = 5.88$ W/m^{2}K

Heat transfer coefficient of outer roof, $h_o = 25$ W/m^{2}K

Indoor room temperature, $T_{\infty,i} = 27$ °C

Ambient temperature, $T_{\infty,o} = 38$°C

Cross-sectional area of the roof, $A = 5 \times 5$ m^2

Problem 3

Incident solar radiation on the roof, $q_{solar} = 1012\ W/m^2$

Absorptivity, $\alpha = 0.4$

Radiation heat transfers between the outer roof surface and the surroundings, and the inner roof surface and room walls are neglected.

(i) With Insulation Case:

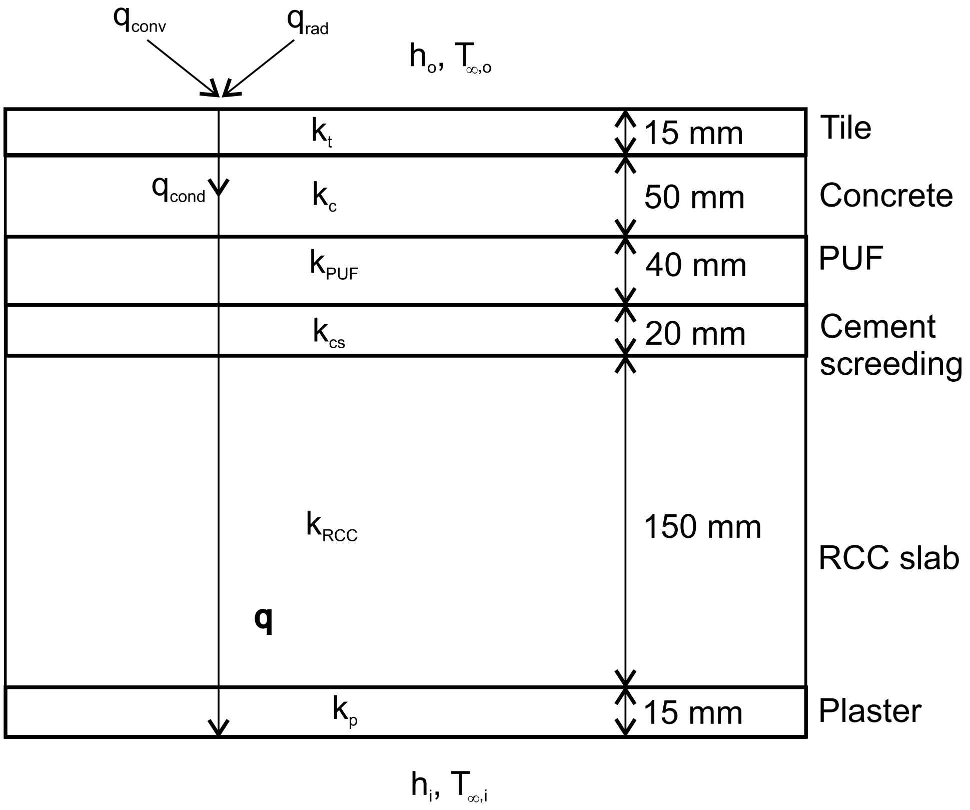

Figure 3.1

Conductive thermal resistance,

$$R_{cond} = \frac{L_t}{k_t A} + \frac{L_c}{k_c A} + \frac{L_{PUF}}{k_{PUF} A} + \frac{L_{cs}}{k_{cs} A} + \frac{L_{RCC}}{k_{RCC} A} + \frac{L_p}{k_p A} \qquad \text{...(3.1)}$$

$$R_{cond} = \frac{0.015}{1.5A} + \frac{0.05}{1.4A} + \frac{0.04}{0.023A} + \frac{0.02}{0.72A} + \frac{0.15}{1.58A} + \frac{0.015}{0.72A}$$

$$R_{cond} = \frac{1.93}{A}\frac{K}{W}$$

Convective thermal resistance:

Inner-roof convective thermal resistance,

$$R_{conv,in} = \frac{1}{h_i A} = \frac{1}{5.88A} = \frac{0.17}{A}\frac{K}{W} \qquad \text{...(3.2)}$$

Outer-roof convective thermal resistance,

$$R_{conv,out} = \frac{1}{h_0 A} = \frac{1}{25A} = \frac{0.04}{A}\frac{K}{W} \qquad \text{...(3.3)}$$

From the heat balance at the outer surface of the roof for steady state condition:

$$q_{rad} + q_{conv} = q_{cond} = q \qquad \text{...(3.4)}$$

$$1012 \times 0.4 \times A + \frac{38 - T_{out,r}}{0.04 / A} = \frac{T_{out,r} - 27}{(1.93 + 0.17) / A}$$

$$404.8 + 950 - 25\ T_{out,r} = 0.48\ T_{out,r} - 12.86$$

$$T_{out,r} = 53.68\ °\mathrm{C}$$

Heat transfer rate,

$$q = \frac{T_{out,r} - \mathrm{T}_{\infty,\mathrm{i}}}{R_{cond} + R_{conv,in}} = \frac{53.68 - 27}{1.93 + 0.17} A = 12.70 \times (5 \times 5) = 317.5\ W \qquad \text{...(3.5)}$$

Heat flux,

$$q'' = \frac{q}{A} = \frac{317.5}{5 \times 5} = 12.70\ W / m^2 \qquad \text{...(3.6)}$$

Temperature of the inner surfaces of the roof assembly, $\mathrm{T}_{\mathrm{in,r}}$

$$q'' = h_i \left(T_{in,r} - T_{\infty,i}\right) \qquad \text{...(3.7)}$$

$$T_{in,r} = \frac{12.70}{5.88} + 27 = 29.16\ °\mathrm{C}$$

(ii) Without Insulation Case:

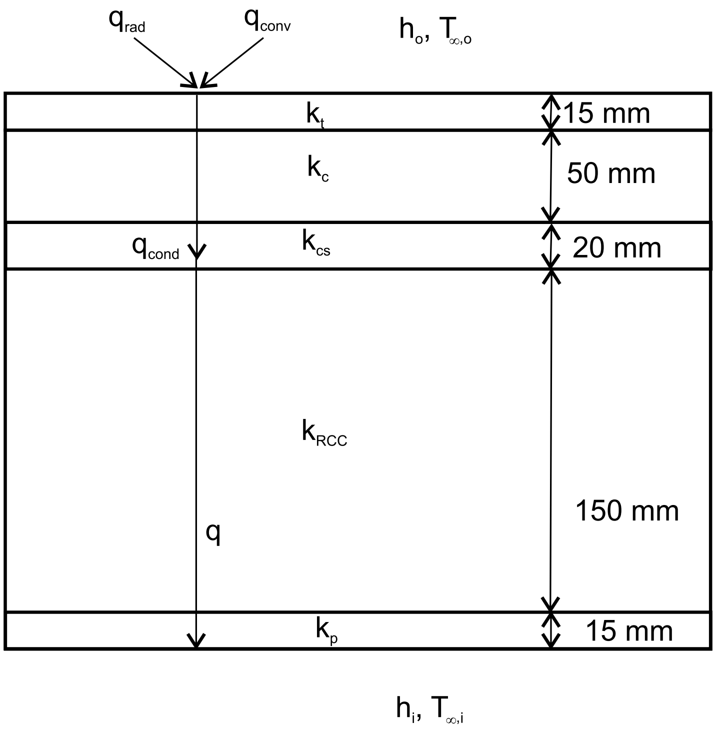

Figure 3.2

Conductive thermal resistance,

$$R_{cond} = \frac{L_t}{k_t A} + \frac{L_c}{k_c A} + \frac{L_{cs}}{k_{cs} A} + \frac{L_{RCC}}{k_{RCC} A} + \frac{L_p}{k_p A} \qquad \text{...(3.8)}$$

$$R_{cond} = \frac{0.015}{1.5A} + \frac{0.05}{1.4A} + \frac{0.02}{0.72A} + \frac{0.15}{1.58A} + \frac{0.015}{0.72A}$$

$$R_{cond} = \frac{0.19}{A}\frac{K}{W}$$

Convective thermal resistance:

Inner and outer-roof convective thermal resistances remain the same as with the insulation case.

From the heat balance at the outer surface of the roof for steady state condition:

$$q_{rad} + q_{conv} = q_{cond} = q \qquad \textbf{...(3.9)}$$

$$1012 \times 0.4 \times A + \frac{38 - T_{out,r}}{0.04 / A} = \frac{T_{out,r} - 27}{(0.19 + 0.17) / A}$$

$$404.8 + 950 - 25\ T_{out,r} = 2.78\ T_{out,r} - 75$$

$$T_{out,r} = 51.47\ °\text{C}$$

Heat transfer rate,

$$q = \frac{T_{out,r} - \text{T}_{\infty,\text{i}}}{R_{cond} + R_{conv,in}} = \frac{51.47 - 27}{0.19 + 0.17} A = 67.97 \times (5 \times 5) = 1699.30\ W \qquad \textbf{...(3.10)}$$

Heat flux,

$$q'' = \frac{q}{A} = \frac{1699.30}{5 \times 5} = 67.97\ W / m^2 \qquad \textbf{...(3.11)}$$

Temperature of the inner surfaces of the roof assembly, $T_{in,r}$

$$q'' = h_i \left(T_{in,r} - T_{\infty,i} \right) \qquad \textbf{...(3.12)}$$

$$T_{in,r} = \frac{67.97}{5.88} + 27 = 38.56\ °\text{C}$$

REMARKS

Results for an uninsulated roof show that the roof contributes significantly to the cooling load with the heat transfer rate of approximately 1.69 kW. It can be observed that the heat transfer inside the room through the roof decreased significantly with the introduction of insulation layer in the roof assembly (1699 W to 317 W). This shows that the cooling load of the building can be reduced significantly by incorporating insulation in the roof assemblies. It can also be noticed that the thermal resistance offered by PUF insulation (1.74/A K/W) contributes significantly to the total thermal resistance (1.93/A K/W) of the wall incorporating insulation. With the incorporation of insulation layer in the roof assembly, the temperature of the inner surface of the roof decreased to 29.16 °C from 38.56 °C.

PROBLEM 4

Building cooling load depends significantly on the outdoor environmental conditions, including solar radiation. It can be reduced by properly designing the building outer walls and roof. Consider a room with all sides exposed to ambient and wall/roof construction similar to the external walls of Aranya Bhawan building located in Jaipur. The outer walls of the building consist of a layer of XPS insulation of 50 mm between two brick layers of 115 mm each. Both sides of the two brick layers are in contact with a 15-mm-thick cement plaster layer. The conductivities of XPS insulation, brick, and plaster are 0.028 W/mK, 0.85 W/mK, and 0.72 W/mK, respectively. The average heat transfer coefficients of inner and outer walls are 7.69 W/m^{2}K and 25.0 W/m^{2}K, respectively. The absorptivity of the exposed surface of the plaster is 0.65.

Roof of the building consists of a 40-mm-thick PUF insulation layer on top of a 20-mm-thick cement screeding. A 15-mm-thick light-coloured terrazzo tile is provided on top of the concrete layer, which is 50-mm thick. At the bottom of the cement screeding layer, there is a layer of 150-mm-thick RCC slab. The bottom-most layer of the roof (inner-most layer) is plaster of 15-mm thickness. The thermal conductivities of terrazzo tile, concrete, PUF insulation, cement screeding, RCC slab, and plaster are 1.50 W/mK, 1.4 W/mK, 0.023 W/mK, 0.72 W/mK, 1.58 W/mK, and 0.72 W/mK, respectively. The average heat transfer coefficients of inner and outer roof sides are 5.88 W/m^{2}K and 25.0 W/m^{2}K, respectively. The absorptivity of the exposed surface of the tile is 0.4.

The room temperature is maintained at 27 °C to achieve thermal comfort. The monthly average ambient temperature and incident solar radiation at all the four walls and the roof of the building for different time of a day, in the month of May in Jaipur, are shown in the table below.

Determine the heat flux, inner and outer surface temperatures of walls and roof at various times of the day. Also plot the variation of the heat flux and inner wall/roof temperature with respect to different wall/roof and time of the day. Discuss the effect of insulation on each of the above parameter.

Table 4.1

Time	Incident solar radiation (in W/m^2)					Outdoor DBT (°C)
	East	West	North	South	Roof	
8.00 a.m.	514.92	94.96	157.19	94.96	313.27	29.96
1.00 p.m.	144.46	230.57	144.46	279.83	1011.95	38.10
4.00 p.m.	133.03	679.52	149.68	140.30	604.98	38.87
9.00 p.m.	0.00	0.00	0.00	0.00	0.00	34.35

SOLUTION

For east wall at 8 a.m.:

(i) With insulation case:

Given:

Thermal conductivity and thickness of plaster, $k_p = 0.72$ W/mK, $L_p = 0.015$ m

Thermal conductivity and thickness of brick, $k_b = 0.85$ W/mK, $L_b = 0.115$ m

Thermal conductivity and thickness of XPS insulation, $k_{XPS} = 0.028$ W/mK, $L_{XPS} = 0.05$ m

Heat transfer coefficient of inner walls, $h_i = 7.69$ W/m^{2}K

Heat transfer coefficient of outer walls, $h_o = 25$ W/m^{2}K

Indoor room temperature, $T_{\infty,i} = 27$ °C

Ambient temperature, $T_{\infty,o} = 29.96$ °C

Incident solar radiation on the wall, $q_{solar} = 514.92$ W/m^2

Absorptivity, $\alpha = 0.65$

Consider cross-sectional area of the roof as A m^2

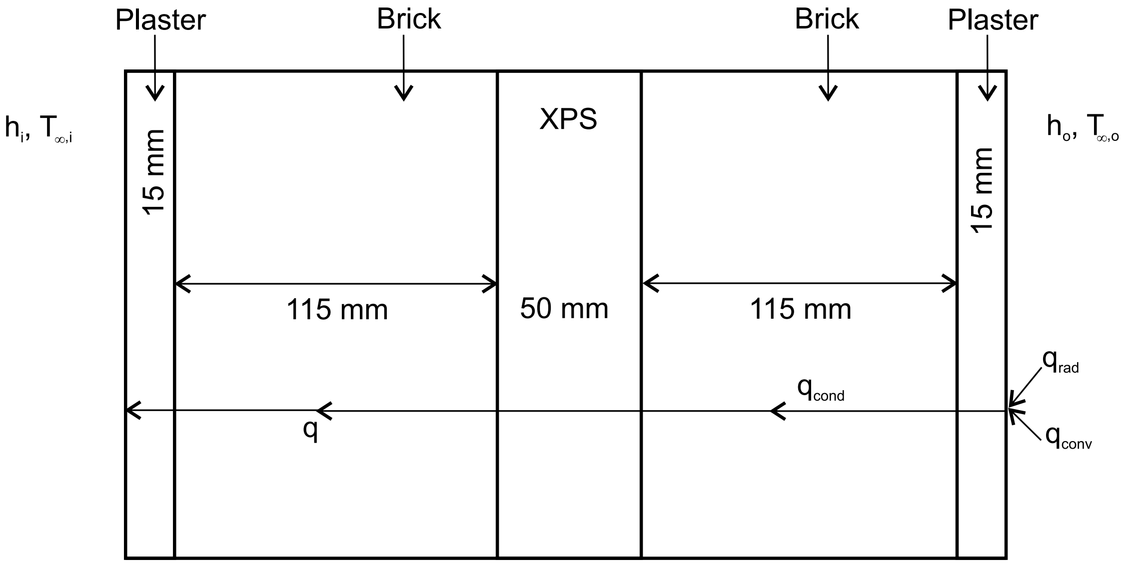

Figure 4.1

Conductive thermal resistance,

$$R_{cond} = \frac{L_p}{k_p A} + \frac{L_b}{k_b A} + \frac{L_{XPS}}{k_{XPS} A} + \frac{L_b}{k_b A} + \frac{L_p}{k_p A} \quad \text{...(4.1)}$$

$$R_{cond} = \frac{0.015}{0.72A} + \frac{0.115}{0.85A} + \frac{0.05}{0.028A} + \frac{0.115}{0.85A} + \frac{0.015}{0.72A}$$

$$R_{cond} = \frac{2.10}{A}\frac{K}{W}$$

Convective thermal resistance:

Inner-wall convective thermal resistance,

$$R_{conv,in} = \frac{1}{h_i A} = \frac{1}{7.69A} = \frac{0.13}{A}\frac{K}{W} \quad \text{...(4.2)}$$

Outer-wall convective thermal resistance,

$$R_{conv,out} = \frac{1}{h_0 A} = \frac{1}{25A} = \frac{0.04}{A}\frac{K}{W} \quad \text{...(4.3)}$$

From the heat balance at the outer surface of the east wall for steady state condition:

$$q_{rad} + q_{conv} = q_{cond} = q \qquad \textbf{...(4.4)}$$

$$514.92 \times 0.65 \times A + \frac{29.96 - T_{out,w}}{0.04 / A} = \frac{T_{out,w} - 27}{(2.10 + 0.13) / A}$$

$$334.70 + 749 - 25\ T_{out,w} = 0.45\ T_{out,w} - 12.11$$

$$T_{out,w} = 43.06\ °\mathrm{C}$$

Heat transfer rate,

$$q = \frac{T_{out,w} - \mathrm{T}_{\infty,\mathrm{i}}}{R_{cond} + R_{conv,in}} = \frac{43.06 - 27}{2.10 + 0.13} A = 7.20A\ W \qquad \textbf{...(4.5)}$$

Heat flux,

$$q'' = \frac{q}{A} = \frac{7.20}{A} = 7.20\ W / m^2 \qquad \textbf{...(4.6)}$$

Temperature of the inner surfaces of the wall assembly, $\mathrm{T}_{\mathrm{in,w}}$

$$q'' = h_i \left(T_{in,w} - T_{\infty,i} \right) \qquad \textbf{...(4.7)}$$

$$T_{in,w} = \frac{7.20}{7.69} + 27 = 27.94\ °\mathrm{C}$$

(ii) Without insulation case:

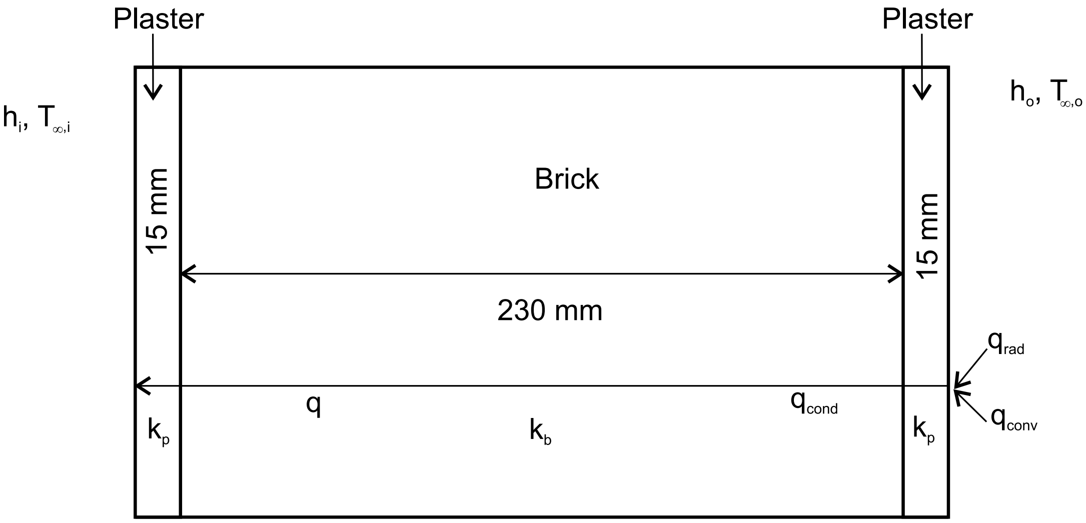

Figure 4.2

Conductive thermal resistance,

$$R_{cond} = \frac{L_p}{k_p A} + \frac{L_b}{k_b A} + \frac{L_p}{k_p A}$$ **...(4.8)**

$$R_{cond} = \frac{0.015}{0.72A} + \frac{0.23}{0.85A} + \frac{0.015}{0.72A}$$

$$R_{cond} = \frac{0.31}{A}\frac{K}{W}$$

Convective thermal resistance:

Inner- and outer-wall convective thermal resistances remain the same as with the insulation case.

From the heat balance at the outer surface of the east wall for steady state condition:

$$q_{rad} + q_{conv} = q_{cond} = q$$ **...(4.9)**

$$514.92 \times 0.65 \times A + \frac{29.96 - T_{out,w}}{0.04 / A} = \frac{T_{out,w} - 27}{(0.31 + 0.13) / A}$$

$$334.70 + 749 - 25\ T_{out,w} = 2.27\ T_{out,w} - 61.36$$

$$T_{out,w} = 41.99\ °\mathrm{C}$$

Heat transfer rate,

$$q = \frac{T_{out,w} - \mathrm{T}_{\infty,\mathrm{i}}}{R_{cond} + R_{conv,in}} = \frac{41.99 - 27}{0.31 + 0.13} A = 33.89 A\ W$$ **...(4.10)**

Heat flux,

$$q'' = \frac{q}{A} = \frac{33.89A}{A} = 33.89\ W / m^2$$ **...(4.11)**

Temperature of the inner surfaces of the wall assembly, $T_{in,w}$

$$q'' = h_i \left(T_{in,w} - T_{\infty,i}\right)$$ **...(4.12)**

$$T_{in,w} = \frac{33.89}{7.69} + 27 = 31.41\ °\mathrm{C}$$

Calculation can be done at other times and walls/roof. Results obtained for various cases are shown in the following tables.

Table 4.2

East wall						
Time	Heat flux q" (W/m^2)		$T_{wall,inner}$(°C)		$T_{wall,outer}$(°C)	
	Without insulation	With insulation	Without insulation	With insulation	Without insulation	With insulation
8.00 a.m.	33.89	7.20	31.41	27.94	41.99	43.06
1.00 p.m.	30.80	6.55	31.00	27.85	40.62	41.59
4.00 p.m.	31.78	6.76	31.13	27.88	41.06	42.06
9.00 p.m.	15.24	3.24	28.98	27.42	33.74	34.22

Table 4.3

West wall						
Time	Heat flux q" (W/m^2)		$T_{wall,inner}$(°C)		$T_{wall,outer}$(°C)	
	Without insulation	With insulation	Without insulation	With insulation	Without insulation	With insulation
8.00 a.m.	11.26	2.39	28.46	27.31	31.98	32.32
1.00 p.m.	35.44	7.54	31.61	27.98	42.68	43.79
4.00 p.m.	61.24	13.02	34.96	28.69	54.09	56.02
9.00 p.m.	15.24	3.24	28.98	27.42	33.74	34.22

Table 4.4

North wall						
Time	Heat flux q" (W/m^2)		$T_{wall,inner}$(°C)		$T_{wall,outer}$(°C)	
	Without insulation	With insulation	Without insulation	With insulation	Without insulation	With insulation
8.00 a.m.	14.61	3.11	28.90	27.40	33.46	33.92
1.00 p.m.	30.80	6.55	31.00	27.85	40.62	41.59
4.00 p.m.	32.68	6.95	31.25	27.90	41.45	42.48
9.00 p.m.	15.24	3.24	28.98	27.42	33.74	34.22

Table 4.5

South wall						
Time	Heat flux q" (W/m^2)		$T_{wall,inner}$(°C)		$T_{wall,outer}$(°C)	
	Without insulation	With insulation	Without insulation	With insulation	Without insulation	With insulation
8.00 a.m.	11.26	2.39	28.46	27.31	31.98	32.33
1.00 p.m.	38.10	8.10	31.95	28.05	43.85	45.05
4.00 p.m.	32.18	6.84	31.18	27.89	41.23	42.24
9.00 p.m.	15.24	3.24	28.98	27.42	33.74	34.22

Table 4.6

Roof with terrazzo tiles						
Time	**Heat flux q" (W/m^2)**		**$T_{wall,inner}$(°C)**		**$T_{wall,outer}$(°C)**	
	Without insulation	**With insulation**	**Without insulation**	**With insulation**	**Without insulation**	**With insulation**
8.00 a.m.	19.96	3.73	30.40	27.63	34.17	34.82
1.00 p.m.	68.34	12.76	38.62	29.17	51.56	53.78
4.00 p.m.	53.96	10.08	36.17	28.71	46.39	48.15
9.00 p.m.	18.40	3.44	30.13	27.58	33.61	34.21

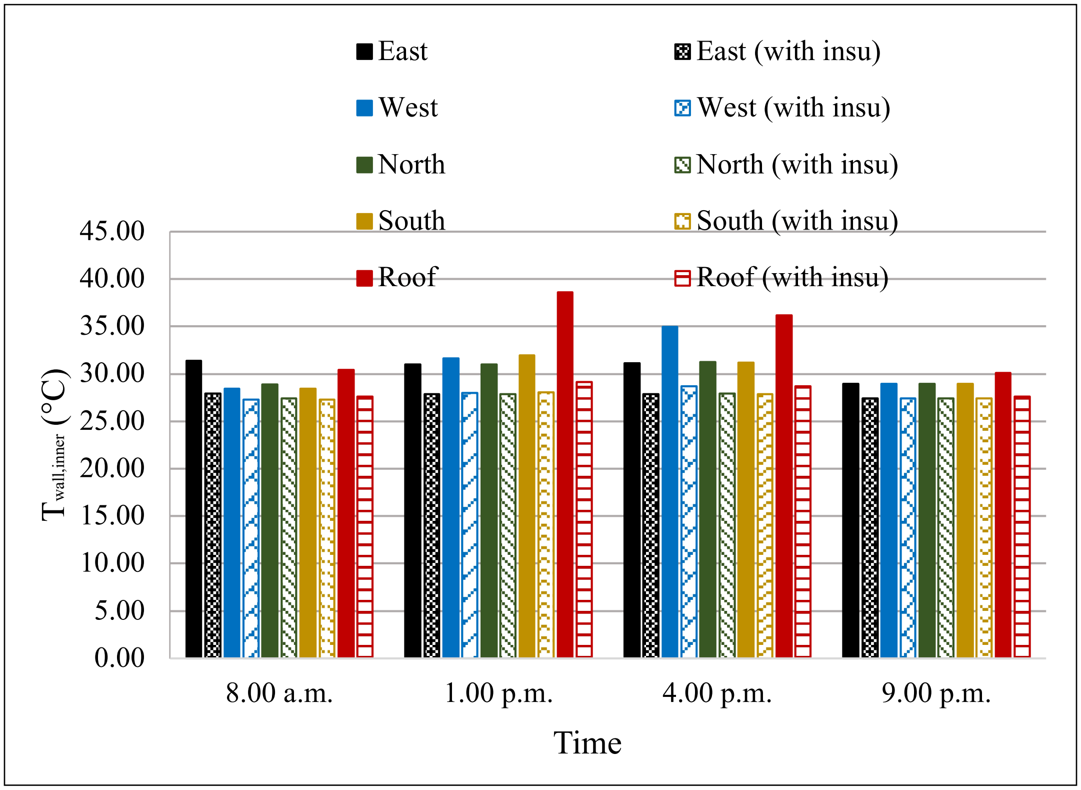

Figure 4.3a

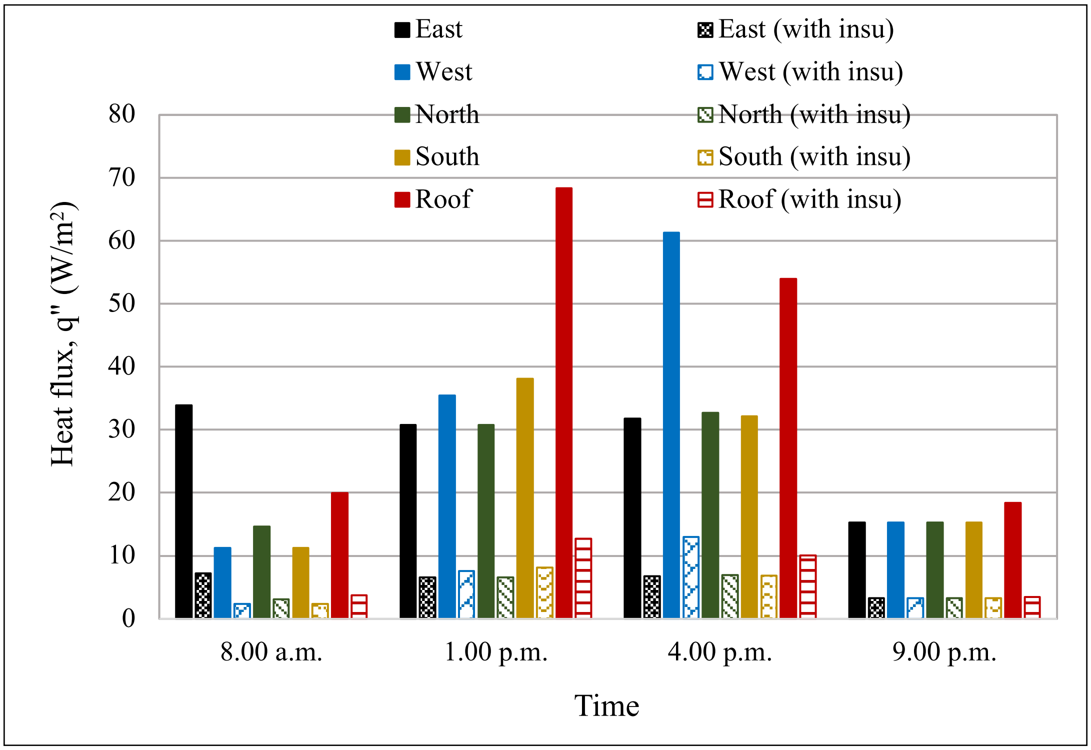

Figure 4.3b

REMARKS

The amount of heat flux governs the inner wall surface temperatures; higher the heat flux, higher the inner wall surface temperature. Heat flux is governed by incident solar radiation and U-value of the wall/roof; higher the incident solar radiation and U-value, higher the heat flux.

With reduction in U-value of the wall and roof by ~80%, similar reduction (~80%) in heat flux is observed.

At 8 a.m., the incident solar radiation is maximum on the east wall and hence, highest heat flux and inner wall surface temperature are noticed for the east wall.

At 1 p.m., the sun is mostly overhead, and maximum solar radiation is falling on the roof. Therefore, heat flux and inner roof surface temperature are found highest for the roof.

At 4 p.m., the sun has moved towards the west and hence, incident solar radiation is maximum on the west wall. Highest heat flux and inner wall surface temperature for west wall are observed.

When there is no solar radiation (9 p.m.), almost no difference in heat flux and inner surface temperatures is observed among different orientations for a construction.

Without insulation, the inner surface temperature can reach as high as 39 °C for roof (at 1 p.m.) and 35 °C for wall (at 4 p.m. on the west wall). With insulation on wall/roof, inner surface temperature of wall/roof can be reduced significantly, helping in achieving better thermal comfort conditions throughout the day in hot summers.

PROBLEM 5

Three roof configurations were proposed/studied for Smart Ghar project, Rajkot, to analyse their effectiveness during summer in hot climate.

- **First roof construction (A normal 3-layer roof design):**
    - Top layer - Cement concrete with waterproofing of thickness 80 mm and exposed surface reflectivity 0.35.
    - Middle layer - RCC slab of thickness 125 mm
    - Bottom layer - Cement plaster of thickness 12 mm.
- **Second roof construction (Roof with reflective tile):**
    - Top layer - China mosaic reflective tile of thickness 20 mm and exposed surface reflectivity 0.70.
    - Second layer - Cement concrete with waterproofing of thickness 80 mm
    - Third layer - RCC slab of thickness 125 mm
    - Bottom layer - Cement plaster of thickness 12 mm.
- **Third roof construction (Painted roof):**
    - Top layer - Cement concrete with waterproofing of thickness 80 mm. Exposed surface of the concrete is coated with a paint of reflectivity 0.85. Thickness of paint coating is negligible.
    - Middle layer - RCC slab of thickness 125 mm
    - Bottom layer - Cement plaster of thickness 12 mm.

The monthly average ambient temperature and incident solar radiation during May month in Rajkot at 1 p.m. are 37 °C and 1036 W/m^2, respectively. The average heat transfer coefficients of inner and outer surfaces are 5.88 W/m^{2}K and 25.0 W/m^{2}K, respectively. The room temperature is maintained at 27 °C to achieve thermal comfort. Find the heat flux through the roof, temperatures at the inner and outer surfaces of the roof assembly considering steady state condition for the three roof configurations. Compare and discuss the results and suggest the best suitable roof configuration.

The thermal conductivities of cement concrete, China mosaic, RCC slab, and cement plaster are 1.40 W/mK, 1.5 W/mK, 1.58 W/mK, and 0.72 W/mK, respectively.

SOLUTION

Given:

Thermal conductivity and thickness of cement concrete, $k_{cs} = 1.4$ W/mK, $L_b = 0.08$ m

Thermal conductivity and thickness of RCC slab, $k_{RCC} = 1.58$ W/mK, $L_b = 0.125$ m

Thermal conductivity and thickness of cement plaster, $k_p = 0.72$ W/mK, $L_p = 0.012$ m

Heat transfer coefficient of inner roof, $h_i = 5.88$ W/m^2 K

Heat transfer coefficient of outer roof, $h_o = 25$ W/m^2 K

Indoor room temperature, $T_{\infty,i}$ = 27 °C

Ambient temperature, $T_{\infty,o}$ = 37 °C

Incident solar radiation on the roof, q_{solar} = 1036 W/m^2

Consider cross-sectional area of the roof as A m^2

(i) Base case (first case):

Absorptivity, $\alpha = 0.65$

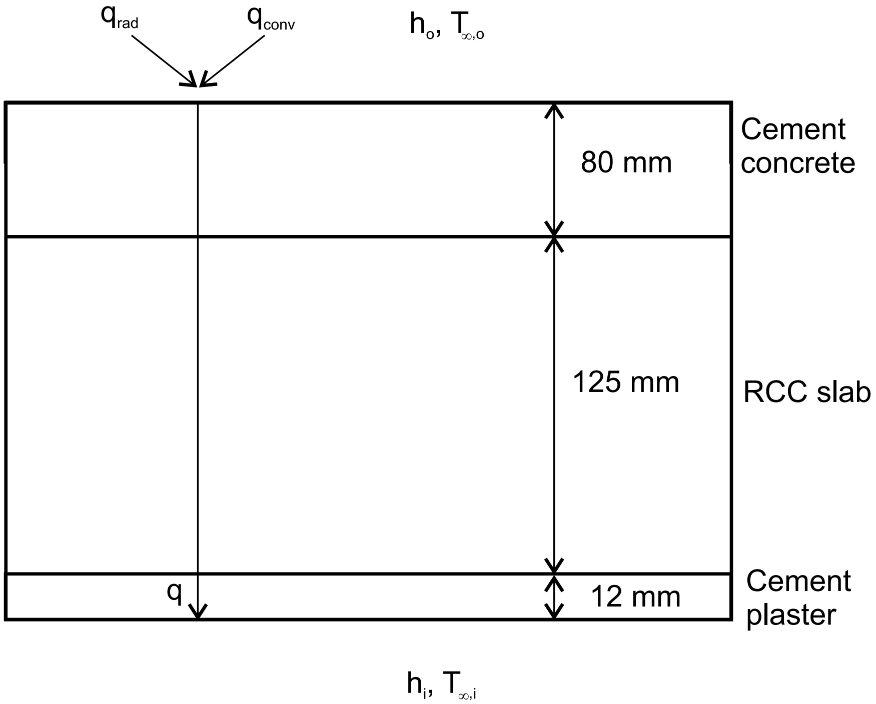

Figure 5.1

Conductive thermal resistance,

$$R_{cond} = \frac{L_{cc}}{k_{cc}A} + \frac{L_{RCC}}{k_{RCC}A} + \frac{L_{cp}}{k_{cp}A} \qquad \text{...(5.1)}$$

$$R_{cond} = \frac{0.08}{1.4A} + \frac{0.125}{1.58A} + \frac{0.012}{0.72A}$$

$$R_{cond} = \frac{0.15}{A}\frac{K}{W}$$

Convective thermal resistance:

Inner-roof convective thermal resistance,

$$R_{conv,in} = \frac{1}{h_i A} = \frac{1}{5.88A} = \frac{0.17}{A}\frac{K}{W} \qquad \text{...(5.2)}$$

Outer-roof convective thermal resistance,

$$R_{conv,out} = \frac{1}{h_0 A} = \frac{1}{25A} = \frac{0.04}{A}\frac{K}{W} \qquad \text{...(5.3)}$$

From the heat balance at the outer surface of the roof for steady state condition:

$$q_{rad} + q_{conv} = q_{cond} = q \qquad \text{...(5.4)}$$

$$1036 \times 0.65 \times A + \frac{37 - T_{out,r}}{0.04/A} = \frac{T_{out,r} - 27}{(0.15 + 0.17)/A}$$

$$673.4 + 925 - 25\ T_{out,r} = 3.12\ T_{out,r} - 84.38$$

$$T_{out,r} = 59.84\ °C$$

Heat transfer rate,

$$q = \frac{T_{out,r} - \mathrm{T}_{\infty,\mathrm{i}}}{R_{cond} + R_{conv,in}} = \frac{59.84 - 27}{0.15 + 0.17} A = 102.62A\ W \qquad \text{...(5.5)}$$

Heat flux,

$$q" = \frac{q}{A} = \frac{102.62A}{A} = 102.62\ W/m^2 \qquad \text{...(5.6)}$$

Temperature of the inner surfaces of the roof assembly, $T_{in,r}$

$$q" = h_i\left(T_{in,r} - T_{\infty,i}\right) \qquad \text{...(5.7)}$$

$$T_{in,r} = \frac{102.62}{5.88} + 27 = 44.45\ °C$$

(ii) Second case (with reflective tile):

One extra China mosaic tile layer is added at the top of the roof of base case.

Thermal conductivity and thickness of tile, $k_t = 1.5$ W/mK, $L_t = 0.02$ m

Now, absorptivity, $\alpha = 0.3$

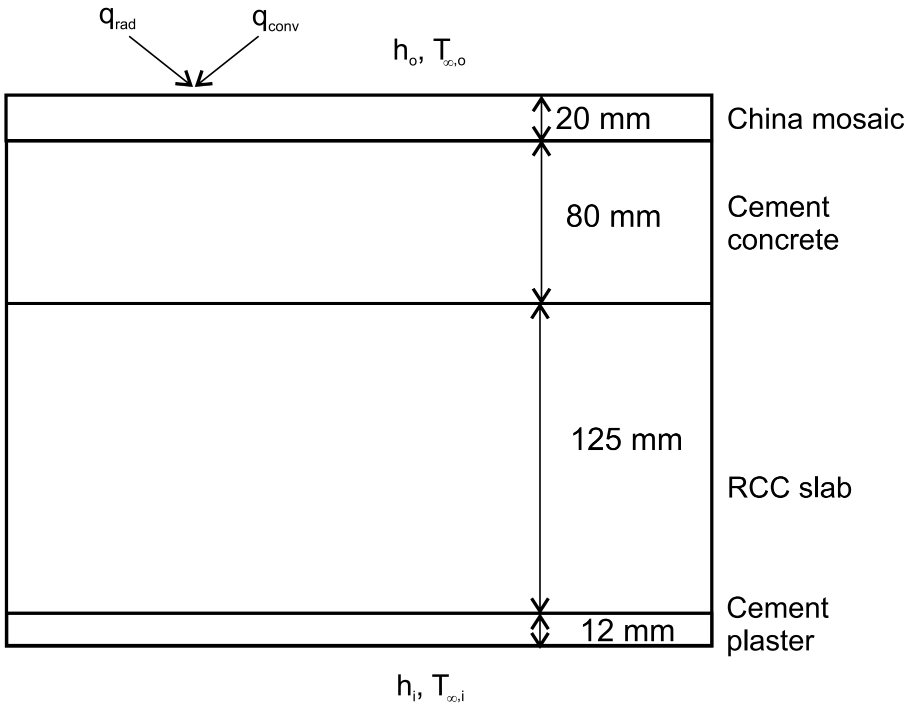

Figure 5.2

Conductive thermal resistance,

$$R_{cond} = \frac{L_t}{k_t A} + \frac{L_{cc}}{k_{cc} A} + \frac{L_{RCC}}{k_{RCC} A} + \frac{L_{cp}}{k_{cp} A} \qquad \textbf{...(5.8)}$$

$$R_{cond} = \frac{0.02}{1.5A} + \frac{0.08}{1.4A} + \frac{0.125}{1.58A} + \frac{0.012}{0.72A}$$

$$R_{cond} = \frac{0.17}{A} \frac{K}{W}$$

Inner- and outer-roof convective thermal resistances remain the same as the base case.

From the heat balance at the outer surface of the roof for steady state condition:

$$q_{rad} + q_{conv} = q_{cond} = q \qquad \textbf{...(5.9)}$$

$$1036 \times 0.3 \times A + \frac{37 - T_{out,r}}{0.04 \big/ A} = \frac{T_{out,r} - 27}{(0.17 + 0.17) \big/ A}$$

$$310.8 + 925 - 25\ T_{out,r} = 2.94\ T_{out,r} - 79.41$$

$$T_{out,r} = 47.07\ °\mathrm{C}$$

Heat transfer rate,

$$q = \frac{T_{out,r} - \mathrm{T}_{\infty,\mathrm{i}}}{R_{cond} + R_{conv,in}} = \frac{47.07 - 27}{0.17 + 0.17} A = 59.03 A\ W \quad \textbf{...(5.10)}$$

Heat flux,

$$q'' = \frac{q}{A} = \frac{59.03 A}{A} = 59.03\ W/m^2 \quad \textbf{...(5.11)}$$

Temperature of the inner surfaces of the roof assembly, $\mathrm{T}_{\mathrm{in,r}}$

$$q'' = h_i\left(T_{in,r} - T_{\infty,i}\right) \quad \textbf{...(5.12)}$$

$$T_{in,r} = \frac{59.03}{5.88} + 27 = 37.04\ °\mathrm{C}$$

(ii) Third case (with reflective paint):

In this case, the roof configuration is the same as the base case. But due to introduction of the reflective paint at the top of the roof, absorptivity (α) decreases to 0.15. Therefore, in this case, all the resistances are same as the base case.

From the heat balance at the outer surface of the roof for steady state condition:

$$q_{rad} + q_{conv} = q_{cond} = q \quad \textbf{...(5.13)}$$

$$1036 \times 0.15 \times A + \frac{37 - T_{out,r}}{0.04/A} = \frac{T_{out,r} - 27}{(0.15 + 0.17)/A}$$

$$155.4 + 925 - 25\ T_{out,r} = 3.12\ T_{out,r} - 84.38$$

$$T_{out,r} = 41.42\ °\mathrm{C}$$

Heat transfer rate,

$$q = \frac{T_{out,r} - \mathrm{T}_{\infty,\mathrm{i}}}{R_{cond} + R_{conv,in}} = \frac{41.42 - 27}{0.15 + 0.17} A = 45.06\ A\ W \quad \textbf{...(5.14)}$$

Heat flux,

$$q'' = \frac{q}{A} = \frac{45.06 A}{A} = 45.06\ W/m^2 \quad \textbf{...(5.15)}$$

Temperature of the inner surfaces of the roof assembly, $T_{in,r}$

$$q'' = h_i\left(T_{in,r} - T_{\infty,i}\right) \qquad \text{...(5.16)}$$

$$T_{in,r} = \frac{45.06}{5.88} + 27 = 34.66\ ^\circ\text{C}$$

REMARKS

As the performance of painted roof is found to be better than the other two roof configurations in terms of heat flux transferred through the roof and inner roof surface temperature as shown in the table below. This is because of the higher reflectivity of the paint as compared to reflective tiles and cement concrete. Results shows that the reflective coating can be a better option as far as the building performance or thermal comfort of building inhabitants are concerned as it reduces the heat transfer through roof significantly without addition of any mass on the roof. Although the reflective tiles and coatings are better options, but their cleanliness, cost, and life should be taken into consideration.

Table 5.1

S. No.	Roof configuration	Heat flux (W/m^2)	Inner roof surface temperature (°C)	Outer roof surface temperature (°C)
1	Normal 3-layer roof design	102.62	44.45	59.84
2	Roof with reflective tile	59.03	37.04	47.07
3	Painted roof	45.06	34.66	41.42

PROBLEM 6

Roof of any building is mostly exposed to solar radiation and hence contributes heavily to the building cooling load compared to various walls of the building. In the present problem, the wall configuration of Aranya Bhawan (Jaipur) and environmental conditions of Jaipur are considered. To decrease the heat transfer through the roof and to minimise the building cooling load, a 40-mm-thick PUF (polyurethane foam) insulation layer is provided on top of the cement screeding of the roof. To reduce the absorption from solar radiation, a layer of 15-mm-thick light-coloured terrazzo tile is provided on top of the concrete layer, which is 50-mm thick. At the bottom of the 20-mm cement screeding layer, there is a layer of 150-mm-thick RCC slab. The bottom-most layer of the roof (inner-most layer) is plaster of 15-mm thickness. The conductivities of terrazzo tile, concrete, PUF insulation, cement screeding, RCC slab, and plaster are 1.50 W/mK, 1.4 W/mK, 0.023 W/mK, 0.72 W/mK, 1.58 W/mK, and 0.72 W/mK, respectively. The average heat transfer coefficients of inner and outer roof sides are 5.88 W/m^{2}K and 25 W/m^{2}K, respectively. The room temperature is maintained at 27 °C to achieve thermal comfort. The ambient temperature and solar radiation during summer (May) in Jaipur at 1.00 p.m. are 38 °C and 1012 W/m^2, respectively. The absorptivity of the exposed surface of the tile is 0.4 and the exposed surface area of the roof is 5 × 5 m.

- Find out the heat transfer rate to the room through the roof and temperatures of the inner and outer surfaces of the roof assembly considering steady state condition.
- To reduce the cooling load further, it was suggested to use cool roof paint/coating at the exposed surface of the roof. The absorptivity of the paint/coating is 0.2. Find out the heat transfer rate to the room through the roof and temperatures of the inner and outer surfaces of the roof assembly considering this surface modification. Compare the result with just the normal terrazzo tile (part 1 of the problem) and suggest if such change is effective or not.

SOLUTION

Given:

Thermal conductivity and thickness of tile, $k_t = 1.5$ W/mK, $L_p = 0.015$ m

Thermal conductivity and thickness of concrete, $k_c = 1.4$ W/mK, $L_b = 0.05$ m

Thermal conductivity and thickness of PUF insulation, $k_{PUF} = 0.023$ W/mK, $L_{XPS} = 0.04$ m

Thermal conductivity and thickness of cement screeding, $k_{cs} = 0.72$ W/mK, $L_b = 0.02$ m

Thermal conductivity and thickness of RCC slab, $k_{RCC} = 1.58$ W/mK, $L_b = 0.15$ m

Thermal conductivity and thickness of plaster, $k_p = 0.72$ W/mK, $L_p = 0.015$ m

Heat transfer coefficient of inner roof, $h_i = 5.88$ W/m^{2}K

Heat transfer coefficient of outer roof, $h_o = 25$ W/m^{2}K

Indoor room temperature, $T_{\infty,i} = 27$ °C

Ambient temperature, $T_{\infty,o} = 38$ °C

Cross-sectional area of the roof, $A = 5 \times 5$ m^2

Incident solar radiation on the roof, $q_{solar} = 1012\ W/m^2$

Absorptivity, $\alpha = 0.4$

(i) Without cool roof coating/painting case:

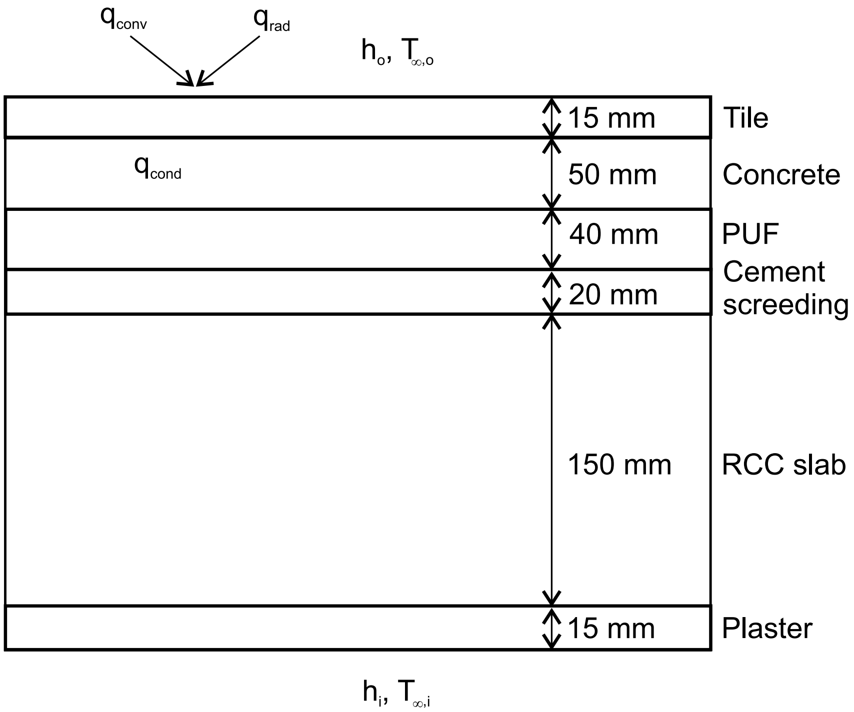

Figure 6.1

Conductive thermal resistance,

$$R_{cond} = \frac{L_t}{k_t A} + \frac{L_c}{k_c A} + \frac{L_{PUF}}{k_{PUF} A} + \frac{L_{cs}}{k_{cs} A} + \frac{L_{RCC}}{k_{RCC} A} + \frac{L_p}{k_p A} \qquad \text{...(6.1)}$$

$$R_{cond} = \frac{0.015}{1.5A} + \frac{0.05}{1.4A} + \frac{0.04}{0.023A} + \frac{0.02}{0.72A} + \frac{0.15}{1.58A} + \frac{0.015}{0.72A}$$

$$R_{cond} = \frac{1.93}{A}\frac{K}{W}$$

Convective thermal resistance:

Inner roof convective thermal resistance,

$$R_{conv,in} = \frac{1}{h_i A} = \frac{1}{5.88A} = \frac{0.17}{A}\frac{K}{W} \qquad \text{...(6.2)}$$

Outer roof convective thermal resistance,

$$R_{conv,out} = \frac{1}{h_0 A} = \frac{1}{25A} = \frac{0.04}{A}\frac{K}{W} \qquad \text{...(6.3)}$$

From the heat balance at the outer surface of the roof for steady state condition:

$$q_{rad} + q_{conv} = q_{cond} = q \quad \textbf{...(6.4)}$$

$$1012 \times 0.4 \times A + \frac{38 - T_{out,r}}{0.04 / A} = \frac{T_{out,r} - 27}{(1.93 + 0.17) / A}$$

$$404.8 + 950 - 25\ T_{out,r} = 0.48\ T_{out,r} - 12.86$$

$$T_{out,r} = 53.68\ °\mathrm{C}$$

Heat transfer rate,

$$q = \frac{T_{out,r} - \mathrm{T}_{\infty,\mathrm{i}}}{R_{cond} + R_{conv,in}} = \frac{53.68 - 27}{1.93 + 0.17} A = 12.70 \times (5 \times 5) = 317.5\ W \quad \textbf{...(6.5)}$$

Heat flux,

$$q" = \frac{q}{A} = \frac{317.5}{5 \times 5} = 12.70\ W / m^2 \quad \textbf{...(6.6)}$$

Temperature of the inner surfaces of the roof assembly, $T_{in,r}$

$$q" = h_i \left(T_{in,r} - T_{\infty,i} \right) \quad \textbf{...(6.7)}$$

$$T_{in,r} = \frac{12.70}{5.88} + 27 = 29.16\ °\mathrm{C}$$

(ii) With cool roof coating/painting case:

In this case, the painting is done on the tile layer. The whole configuration of the layers remains the same, only absorptivity (α) of the outer surface decreases to 0.2.

Conductive thermal resistance,

$$R_{cond} = \frac{L_t}{k_t A} + \frac{L_c}{k_c A} + \frac{L_{PUF}}{k_{PUF} A} + \frac{L_{cs}}{k_{cs} A} + \frac{L_{RCC}}{k_{RCC} A} + \frac{L_p}{k_p A} \quad \textbf{...(6.8)}$$

$$R_{cond} = \frac{0.015}{1.5A} + \frac{0.05}{1.4A} + \frac{0.04}{0.023A} + \frac{0.02}{0.72A} + \frac{0.15}{1.58A} + \frac{0.015}{0.72A}$$

$$R_{cond} = \frac{1.93}{A} \frac{K}{W}$$

Convective thermal resistance:

Inner-roof convective thermal resistance,

$$R_{conv,in} = \frac{1}{h_i A} = \frac{1}{5.88A} = \frac{0.17}{A}\frac{K}{W} \qquad \textbf{...(6.9)}$$

Outer-roof convective thermal resistance,

$$R_{conv,out} = \frac{1}{h_0 A} = \frac{1}{25A} = \frac{0.04}{A}\frac{K}{W} \qquad \textbf{...(6.10)}$$

From the heat balance at the outer surface of the roof for steady state condition:

$$q_{rad} + q_{conv} = q_{cond} = q \qquad \textbf{...(6.11)}$$

$$1012 \times 0.2 \times A + \frac{38 - T_{out,r}}{0.04 / A} = \frac{T_{out,r} - 27}{(1.93 + 0.17) / A}$$

$$202.4 + 950 - 25\ T_{out,r} = 0.48\ T_{out,r} - 12.86$$

$$T_{out,r} = 45.73\ °\mathrm{C}$$

Heat transfer rate,

$$q = \frac{T_{out,r} - \mathrm{T}_{\infty,\mathrm{i}}}{R_{cond} + R_{conv,in}} = \frac{45.73 - 27}{1.93 + 0.17} A = 8.92 \times (5 \times 5) = 222.98\ W \qquad \textbf{...(6.12)}$$

Heat flux,

$$q'' = \frac{q}{A} = \frac{222.98}{5 \times 5} = 8.92 W / m^2 \qquad \textbf{...(6.13)}$$

Temperature of the inner surfaces of the roof assembly, $\mathrm{T}_{\mathrm{in,r}}$

$$q'' = h_i \left(T_{in,r} - T_{\infty,i} \right) \qquad \textbf{...(6.14)}$$

$$T_{in,r} = \frac{8.92}{5.88} + 27 = 28.52\ °\mathrm{C}$$

REMARKS

It can be observed that the heat transfer inside the room through the roof does not change significantly with the introduction of paint/coating over the terrazzo tile. This is mainly due the fact that terrazzo tile itself is a good reflector (60% reflectivity) and the improvement with reflective paint (80% reflectivity) is not very high. However, if compared with a roof having simple plaster finish (30%–35% reflectivity), reflective paints will result in higher difference in the inside roof surface temperature.

PROBLEM 7

Consider an air-conditioned bedroom (4 × 3 m) of a one-family house located in Raipur where the outside air temperature on a typical summer day is 38 °C. The height of the room is 3.5 m. There is one window and one door in the bedroom as shown in Figure 7.1.

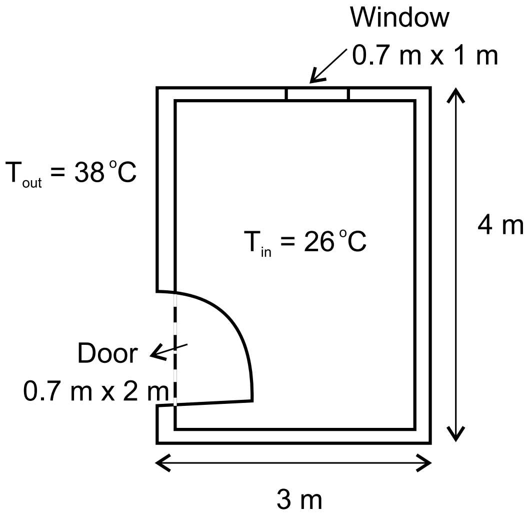

Figure. 7.1

Calculate the cooling load of the bedroom due to air infiltration in the following cases if the desired room temperature is 26 °C:

- When the window and the door are closed, the leakage or air infiltration through the building structure including gaps in windows and the door is 0.5 ACH[1] (reasonably airtight construction).
- When the door of the bedroom is partially opened resulting in air infiltration of 2.0 ACH.

SOLUTION

Given:

Volume of the bedroom, V = 3 × 4 × 3.5 = 42 m^3

Inside air temperature, T_{in} = 26 °C

Outside air temperature, T_{out} = 38 °C

Mean air temperature, T_m = (26 + 38)/2 = 32 °C

Air properties at 32 °C:

Density, ρ = 1.16 kg/m^3

Specific heat, c_p = 1007 J/kg.K

---

[1] ACH (Air changes per hour): It represents the number of times the air in a room/space is replaced in an hour.

Let the air infiltration through the bedroom be Q m^3/s.

(i) Air infiltration through the building structure:

Air changes per hour, ACH = 0.5

$$ACH = \frac{Q \times 3600}{V} \quad \textbf{...(7.1)}$$

$$0.5 = \frac{Q \times 3600}{42}$$

$$Q = 0.00583\left(\mathrm{m}^3/\mathrm{s}\right)$$

Cooling load due to air infiltration considering only the sensible heat exchange,

$$q_{filtration} = \dot{m}c_p\left(T_{out} - T_{in}\right) \quad \textbf{...(7.2)}$$

$$q_{filtration} = \rho Q c_p\left(T_{out} - T_{in}\right) \quad \textbf{...(7.3)}$$

$$q_{filtration} = 1.16 \times 0.00583 \times 1007\left(38 - 26\right)$$

$$q_{filtration} = 81.72\left(W\right)$$

(ii) Air infiltration through partially opened door and building structure:

Air changes per hour, ACH = 2.0

$$ACH = \frac{Q \times 3600}{V} \quad \textbf{...(7.4)}$$

$$2.0 = \frac{Q \times 3600}{42}$$

$$Q = 0.0233\left(m^3/s\right)$$

Cooling load due to the air infiltration,

$$q_{filtration} = \rho Q c_p\left(T_{out} - T_{in}\right) \quad \textbf{...(7.5)}$$

$$q_{filtration} = 1.16 \times 0.0233 \times 1007\left(38 - 26\right)$$

$$q_{filtration} = 326.61\left(W\right)$$

REMARKS

It can be noted that a part of the building cooling load is contributed by air infiltration through cracks and leakages in the windows/doors. The ACH value depends on the quality of construction and the openings in the space. Airtight buildings generally have lower ACH. It can be seen from the above solution that the cooling load due to air infiltration increases as ACH increases. A partially opened bedroom door may lead to approximately 4 times higher (326.62/81.72 ≈ 4.0) cooling load due to air filtration as compared to the case with the door closed. It shows the importance of keeping the air-conditioned spaces as airtight as possible.

PROBLEM 8

Outdoor environmental conditions affect the indoor environment of a building. One external wall (that is exposed to the ambient) of a room contains a layer of XPS insulation of 50 mm in between two brick layers of 115 mm. The brick wall has 15-mm-thick cement plaster on inside and outside surfaces. The conductivities of insulation, brick layer, and cement plaster layer are 0.028 W/mK, 0.85 W/mK, and 0.72 W/mK, respectively. The room with exposed west wall of dimensions 5 × 3.5 m is maintained at 27 °C to achieve thermal comfort. On a typical day in May at 4.00 p.m. in Jaipur, the incident solar radiation on the west wall is 680 W/m^2 and the ambient temperature is 39 °C. The absorptivity of the exposed surface of the plaster is 0.65. The average heat transfer coefficient of inner walls is 7.69 W/m^{2}K. Daily average minimum and maximum wind speeds in the month of May in Jaipur are 12 km/h and 28 km/h, respectively. The heat transfer coefficient (h) can be calculated from the following correlation: $h=5.15\ U^{0.81}$ where U is the wind velocity in m/s. Considering steady state conditions:

1. Determine the convective heat transfer coefficient at the outer side of the wall, the heat transfer rate through the wall, and the inner surface temperature of the wall assembly corresponding to the two wind speeds.
2. Repeat the calculation for a normal construction with 230-mm external brick wall without insulation. Compare and discuss the obtained results with the insulated external wall case.

SOLUTION

(i) With Insulation Case:

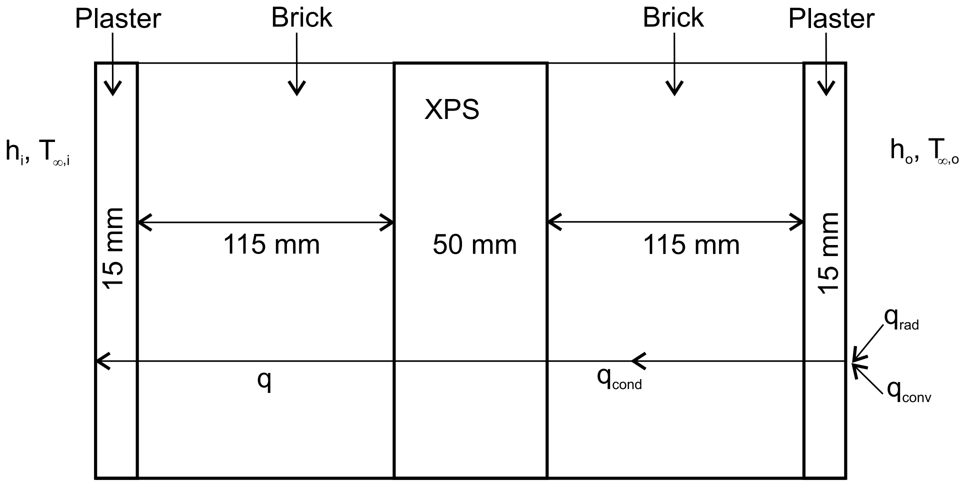

Figure 8.1

Given:

Thermal conductivity and thickness of plaster, $k_p = 0.72$ W/mK, $L_p = 0.015$ m

Thermal conductivity and thickness of brick, $k_b = 0.85$ W/mK, $L_b = 0.115$ m

Thermal conductivity and thickness of XPS insulation, $k_{XPS} = 0.028$ W/mK, $L_{XPS} = 0.05$ m

Heat transfer coefficient of inner walls, $h_i = 7.69$ W/m^2 K

Indoor room temperature, $T_{\infty,i} = 27$ °C

Problem 8

Ambient temperature, $T_{\infty,o} = 39$ °C

Cross-sectional area of the wall, $A = 5 \times 3.5\ m^2$

Incident solar radiation on the wall, $q_{solar} = q_{rad} = 680\ W/m^2$

Absorptivity, $\alpha = 0.65$

Conductive thermal resistance,

$$R_{cond} = \frac{L_p}{k_p A} + \frac{L_b}{k_b A} + \frac{L_{XPS}}{k_{XPS} A} + \frac{L_b}{k_b A} + \frac{L_p}{k_p A} \quad \text{...(8.1)}$$

$$R_{cond} = \frac{0.015}{0.72A} + \frac{0.115}{0.85A} + \frac{0.05}{0.028A} + \frac{0.115}{0.85A} + \frac{0.015}{0.72A}$$

$$R_{cond} = \frac{2.10}{A}\frac{K}{W}$$

Convective thermal resistance:

Inner-wall convective thermal resistance,

$$R_{conv,in} = \frac{1}{h_i A} = \frac{1}{7.69A} = \frac{0.13}{A}\frac{K}{W} \quad \text{...(8.2)}$$

Outer-wall convective thermal resistance:

For velocity; V_{min} = U = 12 km/h = 3.33 m/s:

$$h_o = 5.15 \times U_{10}^{0.81} = 13.66\ W/m^2K \quad \text{...(8.3)}$$

Therefore,

$$R_{conv,out} = \frac{1}{h_0 A} = \frac{1}{13.66A} = \frac{0.073}{A}\frac{K}{W} \quad \text{...(8.4)}$$

From the heat balance at the outer surface of the west wall for steady state condition:

$$q_{rad} + q_{conv} = q_{cond} = q \quad \text{...(8.5)}$$

$$680 \times 0.65 \times A + \frac{39 - T_{out,w}}{0.073/A} = \frac{T_{out,w} - 27}{(2.10 + 0.13)/A}$$

$$442 + 534.25 - 13.70T_{out,w} = 0.45T_{out,w} - 12.11$$

$$T_{out,w} = 69.85\ °C$$

Heat transfer rate,

$$q = \frac{T_{out,w} - \mathrm{T}_{\infty,\mathrm{i}}}{R_{cond} + R_{conv,in}} = \frac{69.85 - 27}{2.10 + 0.13} A = 19.21 \times (5 \times 3.5) = 336.26\ W \qquad \textbf{...(8.6)}$$

Heat flux,

$$q" = \frac{q}{A} = \frac{336.26}{5 \times 3.5} = 19.21\ W/m^2 \qquad \textbf{...(8.7)}$$

Temperature of the inner surfaces of the wall assembly, $\mathrm{T}_{\mathrm{in,w}}$

$$q" = h_i \left(T_{in,w} - T_{\infty,i} \right) \qquad \textbf{...(8.8)}$$

$$T_{in,w} = \frac{19.21}{7.69} + 27 = 29.50\ °\mathrm{C}$$

For velocity; V_{min} = U = 28 km/h = 7.78 m/s:

$$h_o = 5.15 \times U_{10}^{0.81} = 27.13\ W/m^2K \qquad \textbf{...(8.9)}$$

Therefore,

$$R_{conv,out} = \frac{1}{h_0 A} = \frac{1}{27.13A} = \frac{0.037}{A} \frac{K}{W} \qquad \textbf{...(8.10)}$$

From the heat balance at the outer surface of the west wall for steady state condition:

$$q_{rad} + q_{conv} = q_{cond} = q \qquad \textbf{...(8.11)}$$

$$680 \times 0.65 \times A + \frac{39 - T_{out,w}}{0.037/A} = \frac{T_{out,w} - 27}{(2.10 + 0.13)/A}$$

$$T_{out,w} = 54.88\ °\mathrm{C}$$

Heat transfer rate,

$$q = \frac{T_{out,w} - \mathrm{T}_{\infty,\mathrm{i}}}{R_{cond} + R_{conv,in}} = \frac{54.88 - 27}{2.10 + 0.13} A = 12.5 \times (5 \times 3.5) = 218.8\ W \qquad \textbf{...(8.12)}$$

Heat flux,

$$q" = \frac{q}{A} = \frac{218.8}{5 \times 3.5} = 12.5\ W/m^2 \qquad \textbf{...(8.13)}$$

Temperature of the inner surfaces of the wall assembly, $T_{in,w}$

$$q'' = h_i\left(T_{in,w} - T_{\infty,i}\right) \qquad \text{...(8.14)}$$

$$T_{in,w} = \frac{12.5}{7.69} + 27 = 28.62\ ^\circ\text{C}$$

(ii) Without Insulation Case:

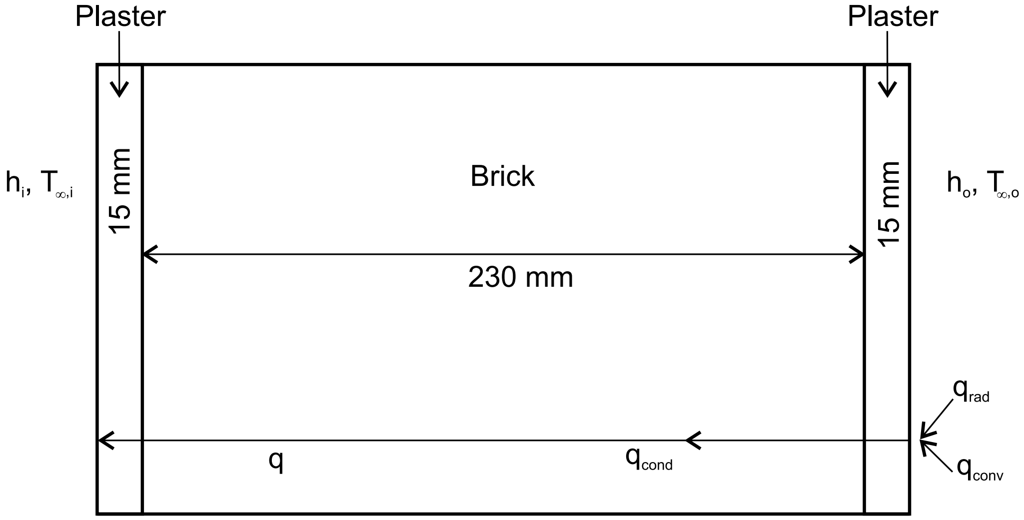

Figure 8.2

Given:

Thermal conductivity and thickness of plaster, $k_p = 0.72$ W/mK, $L_p = 0.015$ m

Thermal conductivity and thickness of brick, $k_b = 0.85$ W/mK, $L_b = 0.230$ m

Heat transfer coefficient of inner walls, $h_i = 7.69$ W/m^2 K

Indoor room temperature, $T_{\infty,i} = 27$ °C

Ambient temperature, $T_{\infty,o} = 39$ °C

Cross-sectional area of the wall, $A = 5 \times 3.5$ m^2

Incident solar radiation on the wall, $q_{solar} = q_{rad} = 680$ W/m^2

Absorptivity, $\alpha = 0.65$

Conductive thermal resistance,

$$R_{cond} = \frac{L_p}{k_p A} + \frac{L_b}{k_b A} + \frac{L_p}{k_p A} \quad \textbf{...(8.15)}$$

$$R_{cond} = \frac{0.015}{0.72A} + \frac{0.23}{0.85A} + + \frac{0.015}{0.72A}$$

$$R_{cond} = \frac{0.31}{A} \frac{K}{W}$$

Convective thermal resistance:

Inner wall convective thermal resistance,

$$R_{conv,in} = \frac{1}{h_i A} = \frac{1}{7.69A} = \frac{0.13}{A} \frac{K}{W} \quad \textbf{...(8.16)}$$

Outer wall convective thermal resistance:

For velocity; V_{min} = U = 12 km/h = 3.33 m/s:

$$h_o = 5.15 \times U_{10}^{0.81} = 13.66 W / m^2 K \quad \textbf{...(8.17)}$$

Therefore,

$$R_{conv,out} = \frac{1}{h_0 A} = \frac{1}{13.66A} = \frac{0.073}{A} \frac{K}{W} \quad \textbf{...(8.18)}$$

From the heat balance at the outer surface of the west wall for steady state condition:

$$q_{rad} + q_{conv} = q_{cond} = q \quad \textbf{...(8.19)}$$

$$680 \times 0.65 \times A + \frac{39 - T_{out,w}}{0.073/A} = \frac{T_{out,w} - 27}{(0.31 + 0.13)/A}$$

$$442 + 534.25 - 13.70\ T_{out,w} = 2.27\ T_{out,w} - 61.36$$

$$T_{out,w} = 64.97\ °\mathrm{C}$$

Heat transfer rate,

$$q = \frac{T_{out,w} - \mathrm{T}_{\infty,\mathrm{i}}}{R_{cond} + R_{conv,in}} = \frac{64.97 - 27}{0.31 + 0.13} A = 86.3 \times (5 \times 3.5) = 1510.27\ W \quad \textbf{...(8.20)}$$

Heat flux,

$$q''=\frac{q}{A}=\frac{1510.27}{5\times 3.5}=86.3\ W/m^2 \qquad \textbf{...(8.21)}$$

Temperature of the inner surfaces of the wall assembly, $T_{\text{in,w}}$

$$q''=h_i\left(T_{in,w}-T_{\infty,i}\right) \qquad \textbf{...(8.22)}$$

$$T_{in,w}=\frac{86.3}{7.69}+27=38.22\ °\text{C}$$

For velocity; V_{min} = U = 28 km/h = 7.78 m/s:

$$h_o=5.15\times U_{10}^{0.81}=27.13\ W/m^2K \qquad \textbf{...(8.23)}$$

Therefore,

$$R_{conv,out}=\frac{1}{h_0A}=\frac{1}{27.13A}=\frac{0.037}{A}\frac{K}{W} \qquad \textbf{...(8.24)}$$

From the heat balance at the outer surface of the west wall for steady state condition:

$$q_{rad}+q_{conv}=q_{cond}=q \qquad \textbf{...(8.25)}$$

$$680\times 0.65\times A+\frac{39-T_{out,w}}{0.037/A}=\frac{T_{out,w}-27}{0.31+0.13/A}$$

$$T_{out,w}=53.15\ °\text{C}$$

Heat transfer rate,

$$q=\frac{T_{out,w}-\text{T}_{\infty,\text{i}}}{R_{cond}+R_{conv,in}}=\frac{53.15-27}{0.31+0.13}A=59.44\times\left(5\times 3.5\right)=1040.21\ W \qquad \textbf{...(8.26)}$$

Heat flux,

$$q''=\frac{q}{A}=\frac{1040.21}{5\times 3.5}=59.44\ W/m^2 \qquad \textbf{...(8.27)}$$

Temperature of the inner surfaces of the wall assembly, $T_{\text{in,w}}$

$$q''=h_i\left(T_{in,w}-T_{\infty,i}\right) \qquad \textbf{...(8.28)}$$

$$T_{in,w}=\frac{59.44}{7.69}+27=34.73\ °\text{C}$$

REMARKS

It can be observed from the tabulated results that wind speed affects both the inner wall temperature and the heat transfer through the wall assembly.The heat flow rate through the wall and the inner surface temperature of the wall decrease when the wind velocity increases because of the increased external convective heat transfer coefficient and increased loss of heat absorbed by radiation to the outside air. The effect of wind speed on inner surface temperature and heat transfer is found to be more significant in the case of uninsulated wall assembly.

Table 8.1

Wall configuration	Wind speed (km/h)	Inner surface temperature of the wall assembly (°C)	Heat flux through the wall assembly (W/m^2)
With insulation	12.0	29.50	19.21
	28.0	28.62	12.50
Without insulation	12.0	38.22	86.30
	28.0	34.73	59.44

PROBLEM 9

Thermocouple is widely used to monitor indoor air temperature as it is inexpensive, interchangeable, and can measure a wide range of temperature. But the main limitation is the error in the measurement caused due to radiation. Consider two roof configurations subjected to the same ambient condition of Rajkot.

- First configuration: Roof assembly of Aranya Bhawan, Jaipur, with top to bottom layers as: terrazzo tile layer (15-mm thick), concrete layer (50-mm thick), PUF insulation layer (40-mm thick), cement screed layer (20-mm thick), RCC slab (150-mm thick), and plaster of thickness 15 mm. The absorptivity of the outer-most layer is 40%.
- Second configuration: Normal roof assembly consisting of three layers. The topmost layer is cement concrete with waterproofing of thickness 80 mm. Middle layer is RCC slab of thickness 125 mm and the bottom layer is cement plaster of thickness 12 mm. The absorptivity of the outer-most layer is 65%.

These two roof configurations are studied in the environment of Rajkot in the month of May when ambient temperature and solar radiation are 37 °C and 1036 W/m^2, respectively. The average heat transfer coefficient of inner and outer sides of the roof are 5.88 W/m^{2}K and 25.0 W/m^{2}K, respectively. To achieve thermal comfort, the room is maintained at 27 °C (actual air temperature) with average indoor air velocity 0.8 m/s. Emissivity of the inner roof surface is 0.85 and that of thermocouple 0.60. Considering thermocouple as a cylindrical body, Nusselt number associated with it can be calculated from the following relation:

$$Nu_{cyl} = 0.3 + \frac{0.62\,\mathrm{Re}^{\frac{1}{2}}\,\mathrm{Pr}^{\frac{1}{3}}}{\left[1+\left(\frac{0.4}{\mathrm{Pr}}\right)^{\frac{2}{3}}\right]^{\frac{1}{4}}}\left[1+\left(\frac{\mathrm{Re}}{282000}\right)^{\frac{5}{8}}\right]^{\frac{4}{5}}$$

Consider the diameter of the thermocouple as 2.3 mm, find out the error in the indoor air temperature measurement with the use of unshielded thermocouple.

Take thermal conductivities of terrazzo tile, concrete, PUF insulation, cement screeding, RCC slab, and plaster as 1.50 W/mK, 1.4 W/mK, 0.023 W/mK, 0.72 W/mK, 1.58 W/mK, and 0.72 W/mK, respectively. Thermal conductivities of cement concrete, RCC slab, and cement plaster are 1.40 W/mK, 1.58 W/mK, and 0.72 W/mK, respectively.

SOLUTION

Given:

Incident solar radiation on the roof, $q_{solar} = q_{rad} = 1036$ W/m^2

Heat transfer coefficient of inner roof, $h_i = 5.88$ W/m^{2}K

Heat transfer coefficient of outer roof, $h_o = 25$ W/m^{2}K

Indoor room temperature, $T_{\infty,i} = 27$ °C

Ambient temperature, $T_{\infty,o} = 37\ °C$

Velocity in the room, $V = 0.8$ m/s

Emissivity of the inner roof surface, $\varepsilon_{roof} = 0.85$

Emissivity of the thermocouple, $\varepsilon_{ther} = 0.6$

Diameter of the thermocouple, $d = 2.3$ mm

(i) Case-I: First configuration:

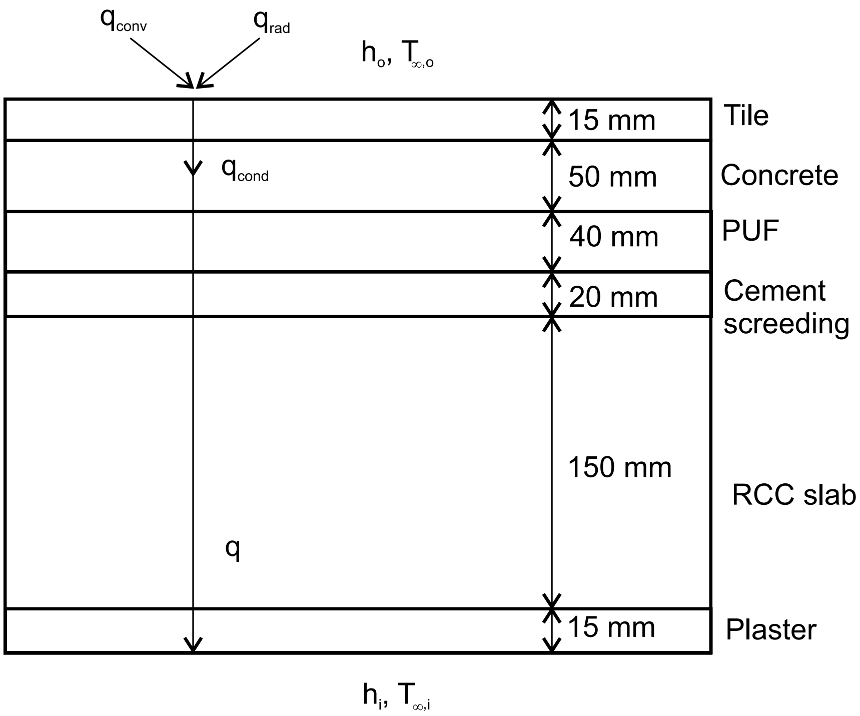

Figure 9.1

Given:

Thermal conductivity and thickness of tile, $k_t = 1.5$ W/mK, $L_t = 0.015$ m

Thermal conductivity and thickness of concrete, $k_c = 1.4$ W/mK, $L_c = 0.05$ m

Thermal conductivity and thickness of PUF insulation, $k_{PUF} = 0.023$ W/mK, $L_{PUF} = 0.04$ m

Thermal conductivity and thickness of cement screeding, $k_{cs} = 0.72$ W/mK, $L_{cs} = 0.02$ m

Thermal conductivity and thickness of RCC slab, $k_{RCC} = 1.58$ W/mK, $L_{RCC} = 0.15$ m

Thermal conductivity and thickness of plaster, $k_p = 0.72$ W/mK, $L_p = 0.015$ m

Absorptivity, $\alpha = 0.4$

Problem 9

Let the surface area of the roof be A.

Conductive thermal resistance,

$$R_{cond} = \frac{L_t}{k_t A} + \frac{L_c}{k_c A} + \frac{L_{PUF}}{k_{PUF} A} + \frac{L_{cs}}{k_{cs} A} + \frac{L_{RCC}}{k_{RCC} A} + \frac{L_p}{k_p A} \qquad \textbf{...(9.1)}$$

$$R_{cond} = \frac{0.015}{1.5A} + \frac{0.05}{1.4A} + \frac{0.04}{0.023A} + \frac{0.02}{0.72A} + \frac{0.15}{1.58A} + \frac{0.015}{0.72A}$$

$$R_{cond} = \frac{1.93}{A} \frac{K}{W}$$

Convective thermal resistance:

Inner-roof convective thermal resistance,

$$R_{conv,in} = \frac{1}{h_i A} = \frac{1}{5.88A} = \frac{0.17}{A} \frac{K}{W} \qquad \textbf{...(9.2)}$$

Outer-roof convective thermal resistance,

$$R_{conv,out} = \frac{1}{h_0 A} = \frac{1}{25A} = \frac{0.04}{A} \frac{K}{W} \qquad \textbf{...(9.3)}$$

From the heat balance at the outer surface of the roof for steady state condition:

$$q_{rad} + q_{conv} = q_{cond} = q \qquad \textbf{...(9.4)}$$

$$1036 \times 0.4 \times A + \frac{37 - T_{out,r}}{0.04 / A} = \frac{T_{out,r} - 27}{1.93 + 0.17 / A}$$

$$414.4 + 925 - 25\ T_{out,r} = 0.48\ T_{out,r} - 12.86$$

$$T_{out,r} = 53.07\ °\mathrm{C}$$

Heat transfer rate,

$$q = \frac{T_{out,r} - \mathrm{T}_{\infty,\mathrm{i}}}{R_{cond} + R_{conv,in}} = \frac{53.07 - 27}{1.93 + 0.17} A = 12.41\ AW \qquad \textbf{...(9.5)}$$

Heat flux,

$$q'' = \frac{q}{A} = \frac{12.41A}{A} = 12.41\ W/m^2 \qquad \textbf{...(9.6)}$$

Temperature of the inner surfaces of the roof assembly, $T_{in,r}$

$$q'' = h_i \left(T_{in,r} - T_{\infty,i} \right) \qquad \textbf{...(9.7)}$$

$$T_{in,r} = \frac{12.41}{5.88} + 27 = 29.11 \text{ °C}$$

Thermophysical properties of air at temperature, (27+29.11)/2 = 28.06 °C:

Thermal conductivity, k = 0.0257 W/mK

Kinematic viscosity, $\nu = 1.59 \times 10^{-5}$ m^2/s

Prandtl number, Pr = 0.7287

Reynolds number, $Re_d = Vd/\nu = 115.72$

Nusselt number:

$$Nu_{cyl} = 0.3 + \frac{0.62 \operatorname{Re}^{\frac{1}{2}} \operatorname{Pr}^{\frac{1}{3}}}{\left[1 + \left(\frac{0.4}{\operatorname{Pr}} \right)^{\frac{2}{3}} \right]^{\frac{1}{4}}} \left[1 + \left(\frac{\operatorname{Re}}{282000} \right)^{\frac{5}{8}} \right]^{\frac{4}{5}} \qquad \textbf{...(9.8)}$$

$$Nu_{cyl} = 5.61$$

Again,

$$Nu_{cyl} = \frac{hd}{k} = 5.61 \qquad \textbf{...(9.9)}$$

$$h = \frac{Nu_{cyl} \times k}{d} = 62.68 \; W / m^2 K$$

From the heat balance of the thermocouple with following assumptions:

- Inner surfaces of all side walls and the floor are at the same temperature as that of the inner surface of the roof.
- Surface area of the thermocouple can be assumed to be very small compared to the enclosure area. So, it can be assumed as a case of radiation exchange between a small object kept in a large enclosure. (Where q_{rad} calculation was done).
- Let the temperature of the thermocouple (measured temperature) be$T_{ther.}$

$$q_{conv} = q_{rad} \qquad \text{...(9.10)}$$

$$h\left(T_{\infty,i} - T_{ther}\right) = \varepsilon_{ther}\sigma\left(T_{ther}^4 - T_{in,r}^4\right)$$

$$62.68\left(300 - T_{ther}\right) = 0.6 \times 5.67 \times 10^{-8}\left(T_{ther}^4 - 302.11^4\right)$$

$$3.4 \times 10^{-8} T_{ther}^4 + 62.68 T_{ther} - 19087.4 = 0$$

$$T_{ther} = 300.12K = 27.12\ °\mathrm{C}$$

Error in measurement = Measured temperature – Actual room temperature = 27.12 – 27 = 0.12 °C

Percentage error in measurement = (0.12/27) × 100 = 0.44%

(ii) Case-II: Second configuration:

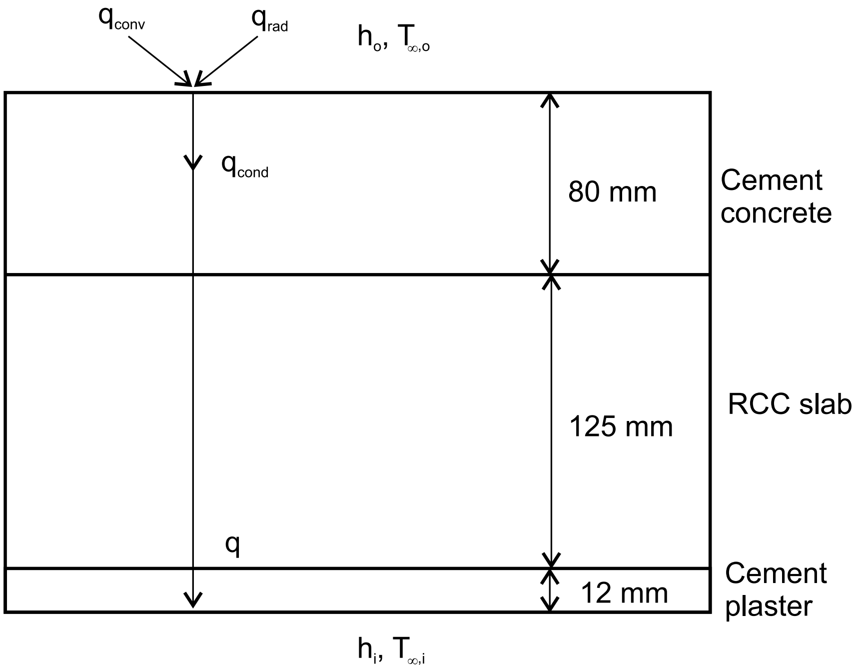

Figure 9.2

Given:

Thermal conductivity and thickness of cement concrete, k_{cc} = 1.4 W/mK, L_{cc} = 0.08 m

Thermal conductivity and thickness of RCC slab, k_{RCC} = 1.58 W/mK, L_{RCC} = 0.125 m

Thermal conductivity and thickness of cement plaster, k_p = 0.72 W/mK, L_p = 0.012 m

Absorptivity, α = 0.65

Let surface area of the roof is A.

Conductive thermal resistance,

$$R_{cond} = \frac{L_{cc}}{k_{cc}A} + \frac{L_{RCC}}{k_{RCC}A} + \frac{L_{cp}}{k_{cp}A} \quad \textbf{...(9.11)}$$

$$R_{cond} = \frac{0.08}{1.4A} + \frac{0.125}{1.58A} + \frac{0.012}{0.72A}$$

$$R_{cond} = \frac{0.15}{A}\frac{K}{W}$$

Convective thermal resistance:

Inner roof convective thermal resistance,

$$R_{conv,in} = \frac{1}{h_i A} = \frac{1}{5.88A} = \frac{0.17}{A}\frac{K}{W} \quad \textbf{...(9.12)}$$

Outer roof convective thermal resistance,

$$R_{conv,out} = \frac{1}{h_0 A} = \frac{1}{25A} = \frac{0.04}{A}\frac{K}{W} \quad \textbf{...(9.13)}$$

From the heat balance at the outer surface of the roof for steady state condition:

$$q_{rad} + q_{conv} = q_{rad} = q \quad \textbf{...(9.14)}$$

$$1036 \times 0.65 \times A + \frac{37 - T_{out,r}}{0.04/A} = \frac{T_{out,r} - 27}{(0.15 + 0.17)/A}$$

$$673.4 + 925 - 25\ T_{out,r} = 3.12\ T_{out,r} - 84.38$$

$$T_{out,r} = 59.84\ °\mathrm{C}$$

Heat transfer rate,

$$q = \frac{T_{out,r} - \mathrm{T}_{\infty,\mathrm{i}}}{R_{cond} + R_{conv,in}} = \frac{59.84 - 27}{0.15 + 0.17} A = 102.62A\ W \quad \textbf{...(9.15)}$$

Heat flux,

$$q'' = \frac{q}{A} = \frac{102.62A}{A} = 102.62\ W/m^2 \quad \textbf{...(9.16)}$$

Temperature of the inner surfaces of the roof assembly, $T_{in,r}$

$$q'' = h_i\left(T_{in,r} - T_{\infty,i}\right) \quad \text{...(9.17)}$$

$$T_{in,r} = \frac{102.62}{5.88} + 27 = 44.45 \text{ °C}$$

Thermophysical properties of air at temperature, (27 + 44.5)/2 = 35.72 °C:

Thermal conductivity, k = 0.026 W/mK

Kinematic viscosity, $\nu = 1.66 \times 10^{-5}$ m^2/s

Prandtl number, Pr = 0.7266

Reynolds number, $Re_d = Vd/\nu = 110.84$

Nusselt number:

$$Nu_{cyl} = 0.3 + \frac{0.62\,\mathrm{Re}^{\frac{1}{2}}\,\mathrm{Pr}^{\frac{1}{3}}}{\left[1+\left(\frac{0.4}{\mathrm{Pr}}\right)^{\frac{2}{3}}\right]^{\frac{1}{4}}}\left[1+\left(\frac{\mathrm{Re}}{282000}\right)^{\frac{5}{8}}\right]^{\frac{4}{5}} \quad \text{...(9.18)}$$

$$Nu_{cyl} = 5.49$$

Again,

$$Nu_{cyl} = \frac{hd}{k} = 5.49 \quad \text{...(9.19)}$$

$$h = \frac{Nu_{cyl} \times k}{d} = 62.06\ W/m^2K$$

Let the temperature of the thermocouple (measured temperature) is T_{ther}. Heat balance of the thermocouple:

$$q_{conv} = q_{rad} \quad \text{...(9.20)}$$

$$h\left(T_{\infty,i} - T_{ther}\right) = \varepsilon_{ther}\sigma\left(T_{ther}^4 - T_{in,r}^4\right)$$

$$62.06\left(300 - T_{ther}\right) = 0.6 \times 5.67 \times 10^{-8}\left(T_{ther}^4 - 317.45^4\right)$$

$$3.4 \times 10^{-8} T_{ther}^4 + 62.06\ T_{ther} - 18963.49 = 0$$

$$T_{ther} = 301.06K = 28.06 \text{ °C}$$

Error in measurement = 28.06 – 27 = 1.06 °C

Percentage error in measurement = (1.06/27) × 100 = 3.92%

REMARKS

It can be observed that for normal roof configuration, error in the temperature measurement of thermocouple is higher (3.92%) compared to the roof configuration of Aranya Bhawan (0.44%). This is because there is more radiative heat transfer between the thermocouple and the roof due to higher inner surface temperature of roof in the case of normal roof construction. Hence, the error of thermocouple is dependent on the temperature difference between indoor wall/roof surface temperature and the air temperature. Lower temperature difference results in lower error in the measurement. Apart from this, the error in temperature measurement also depends on the emissivity (ε) of the thermocouple and the associated convective heat transfer coefficient (h). Increase in h value decreases the error and decrease in ε decreases the error in the measurement. Considering the error associated with the temperature measurement by thermocouples due to radiation heat transfer, it is recommended to use shielded thermocouples for the measurement of room air temperature.

PROBLEM 10

A flat roof building of size 25 m × 60 m and 4 m height is to be built in Raipur. Monthly average (May month) values of incident solar radiation on different walls of the building located in Raipur throughout the day are given in Table 10.1. Based on the given monthly average data, suggest the appropriate orientation of the building among the two orientations (A and B) shown in Fig. 10.1 to ensure lower exposure of building to the solar radiation.

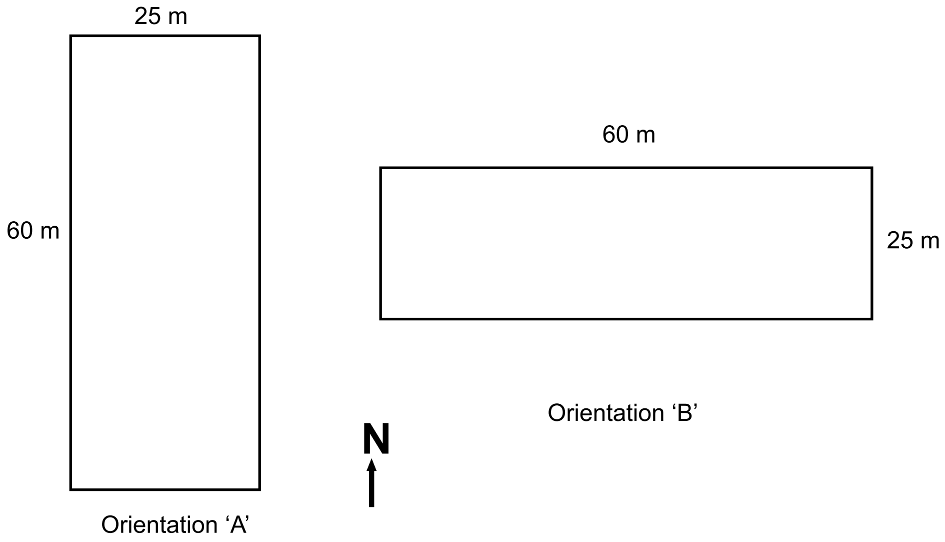

Figure 10.1: Two building orientations

Table 10.1: Monthly average incident solar radiation, I_{solar} (W/m^2) for the month of May on differently oriented vertical walls of a building located at Raipur

Time (h)	**Forenoon (a.m.)**							**Noon**	**Afternoon (p.m.)**						
	5	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**
East wall	0	252	516	643	660	576	414	205	155	156	142	112	66	19	0
West wall	0	26	81	124	151	163	162	152	307	472	560	529	358	115	0
North wall	0	102	189	214	204	180	166	152	156	165	171	170	127	47	0
South wall	0	26	81	124	151	169	188	190	185	170	142	112	66	19	0

SOLUTION

Total incident solar radiation on the east wall throughout the day:

$I_{solar,east} = 252 + 516 + 643 + 660 + 576 + 414 + 205 + 155 + 142 + 112 + 66 + 19$

$I_{solar,east} = 3918\ \text{W/m}^2$

Similarly, the total incident solar radiation on the remaining walls:

$I_{solar,west} = 3201\ \text{W/m}^2$

$I_{solar,north} = 2042\ \text{W/m}^2$

$I_{solar,south} = 1623\ \text{W/m}^2$

(i) In case of building orientation 'A':

Area of the east and west walls, $A_{\text{east}} = A_{\text{west}} = 60 \times 4 = 240\ \text{m}^2$

Area of the north and south walls, $A_{\text{north}} = A_{\text{south}} = 25 \times 4 = 100\ \text{m}^2$

Total solar radiation received by the four walls:

$q_{\text{total}} = \Sigma(I \times A) = 3918 \times 240 + 3201 \times 240 + 2042 \times 100 + 1623 \times 100$

$\boldsymbol{q_{\text{total}} = 2{,}074{,}987\ \text{W}}$

(i) In case of building orientation 'B':

Area of the east and west walls, $A_{\text{east}} = A_{\text{west}} = 25 \times 4 = 100\ \text{m}^2$

Area of the north and south walls, $A_{\text{north}} = A_{\text{south}} = 60 \times 4 = 240\ \text{m}^2$

Total solar radiation received by the four walls:

$q_{\text{total}} = \Sigma(I \times A) = 3918 \times 100 + 3201 \times 100 + 2042 \times 240 + 1623 \times 240$

$\boldsymbol{q_{\text{total}} = 1{,}591{,}504\ \text{W}}$

REMARKS

Roof of the building will receive same amount of solar radiation in both the orientations as it is a flat roof. Please note a large variation in the hourly incident solar radiation on differently oriented walls. Further, the walls facing east and west receives larger amount of radiation compared to walls facing south and north. It can be observed from above that the building will receive overall higher amount of solar radiation (i.e., 2,074,987 W) in case of orientation A (larger area of walls facing east and west) compared to orientation B (1,591,504 W). Hence, orientation 'B' (smaller area of walls facing east and west) is recommended for receiving lower solar radiation (consequently lower heat load for air-conditioning during the summer month of May) based on the climate data of Raipur. It is to be noted here that the effect of only solar radiation is considered here.

PROBLEM 11

Windows provide daylight and natural ventilation inside the room. However, it contributes significantly to the building cooling load compared to the external walls and roof of the building. Consider a room of a building located in Jaipur with west-facing external wall incorporating a window. The thickness and thermal conductivity of the glass window is 3.2 mm and 0.92 W/mK, respectively. Transmissivity and absorptivity of the glass at 60° incident angle are 0.75 and 0.11, respectively. The average incident solar radiation on the west wall is 680 W/m^2 and the ambient temperature is 39 ºC in the summer month of May in Jaipur at 4.00 p.m. The room is maintained at 27 °C to achieve thermal comfort. The average heat transfer coefficients associated with the inner and outer surfaces of the window are 7.69 W/m^{2}K and 25 W/m^{2}K, respectively.

- Calculate the heat flux through the window and temperatures of the inner and outer surfaces of the window considering steady state condition.
- Calculate the heat flux through the window based on the solar heat gain coefficient (SHGC) and compare the results.

SOLUTION

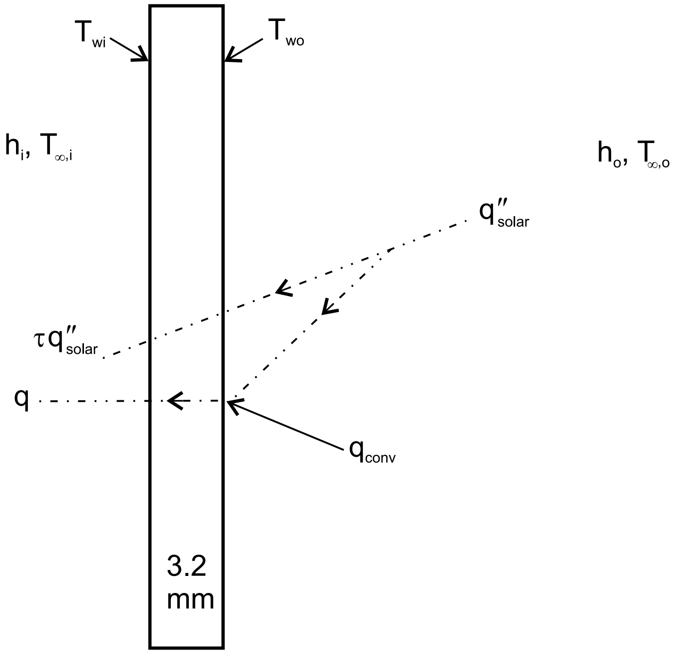

Figure 11.1

Given:

Thermal conductivity and thickness of the window glass, k_g = 0.92 W/mK, L_g = 0.0032 m

Heat transfer coefficient of inner window glass, h_i = 7.69 W/m^{2}K

Heat transfer coefficient of outer window glass, h_o = 25 W/m^{2}K

Indoor room temperature, $T_{\infty, i}$ = 27 °C

Ambient temperature, $T_{\infty,o}$ = 39 °C

Incident solar radiation on the wall, q''_{solar} = 680 W/m^2

For 60° incidence angle,

Absorptivity, α = 0.11

Transmissivity, τ = 0.75

From the heat balance at the outer surface of the window:

Let cross-sectional area of the window be is A m^2. Neglecting the radiation heat transfer between the outer surface of the window and the surroundings, and the inner surface of the window and room walls/roof:

$$q_{rad} + q_{conv} = q_{cond} = q \tag*{...(11.1)}$$

$$\alpha \times q''_{solar} \times A + h_o\left(T_{\infty,o} - T_{w,o}\right)A = \frac{T_{w,o} - T_{\infty,i}}{R_{cond} + R_{conv,in}}A$$

$$0.11 \times 680A + 25\left(39 - T_{w,o}\right)A = \frac{T_{w,o} - 27}{\frac{0.0032}{0.92} + \frac{1}{7.69}}A$$

$$T_{w,o} = 38.54 \text{ °C}$$

Heat conducted through the window,

$$q'' = \frac{38.54 - 27}{\frac{0.1335}{A}A} = 86.44\ W/m^2 \tag*{...(11.2)}$$

Heat transmitted through the window,

$$q''_{trans} = \tau \times q''_{solar} = 510\ W/m^2 \tag*{...(11.3)}$$

Total heat flux,

$$q''_{total} = q'' + q''_{trans} = 86.44 + 510 = 596.44\ W/m^2 \tag*{...(11.4)}$$

Temperature at the inner surfaces of the window, $T_{w,in}$

$$q'' = h_i\left(T_{w,i} - T_{\infty,i}\right) \tag*{...(11.5)}$$

$$T_{w,i} = 38.24 \text{ °C}$$

Heat flux calculation using SHGC:

Heat flux,

$$q''_{total} = U\left(T_{\infty,o} - T_{\infty,i}\right) + SHGC \times q''_{solar} \qquad \text{...(11.6)}$$

where, U is the overall heat transfer coefficient of the glass and SHGC is the solar heat gain coefficient.

Here,

$$\frac{1}{U} = \frac{1}{h_i} + \frac{L_g}{k_g} + \frac{1}{h_o} = \frac{1}{7.69} + \frac{0.0032}{0.92} + \frac{1}{25} \qquad \text{...(11.7)}$$

$$U = 5.76 W/m^2K$$

SHGC:

$$SHGC = \tau + \alpha \frac{U}{h_o} = 0.75 + 0.11\frac{5.76}{25} = 0.78 \qquad \text{...(11.8)}$$

Therefore,

$$q''_{total} = 5.76(39-27) + 0.78 \times 680 = 599.52 \ W/m^2 \qquad \text{...(11.9)}$$

REMARKS

It can be observed from the calculations that in the case of windows, significant portion of the heat is transferred through it by transmission. The heat flux due to transmission is 510 W/m^2 whereas the heat transferred due to conduction is just 86.44 W/m^2. It is also observed that the temperature difference between the inner and outer surfaces of the window is just 0.3 °C. Thus, conduction resistance of the window glass may be neglected in general and the window can be considered as a lumped system. Further, the heat flux calculated based on SHGC is found to be 599.52 W/m^2 compared to 596.44 W/m^2 obtained based on normal heat balance calculations. This shows that both the methods can be used to determine the extent of heat transfer through windows.

PROBLEM 12

Consider a double-pane window having two glasses of 3.2-mm thickness each, separated by an air gap of 6-mm thickness. Transmittivity and thermal conductivity of both the glasses are 0.75 and 0.92 W/mK. Absorptivities of the outer glass and inner glass at 60° incident angle are 0.12 and 0.08, respectively. Thermal conductivity of air is 0.026 W/mK. The window is located at the west wall of a building located in Jaipur. The average incident solar radiation on the west wall is 680 W/m^2 and the ambient temperature is 39 ºC in Jaipur at 4.00 p.m. in a typical summer month. The room is maintained at 27 °C to achieve thermal comfort. The average heat transfer coefficients associated with the inner and outer surfaces of the window are 7.69 W/m^2K and 25.0 W/m^2K, respectively. Considering only conduction heat transfer through the air between the two panes of window:

(i) Calculate the heat flux through the window and temperatures of the inner and outer surfaces of the double-pane window considering steady state condition.

(ii) Calculate the heat flux through the window based on the solar heat gain coefficient (SHGC) and compare the results.

SOLUTION

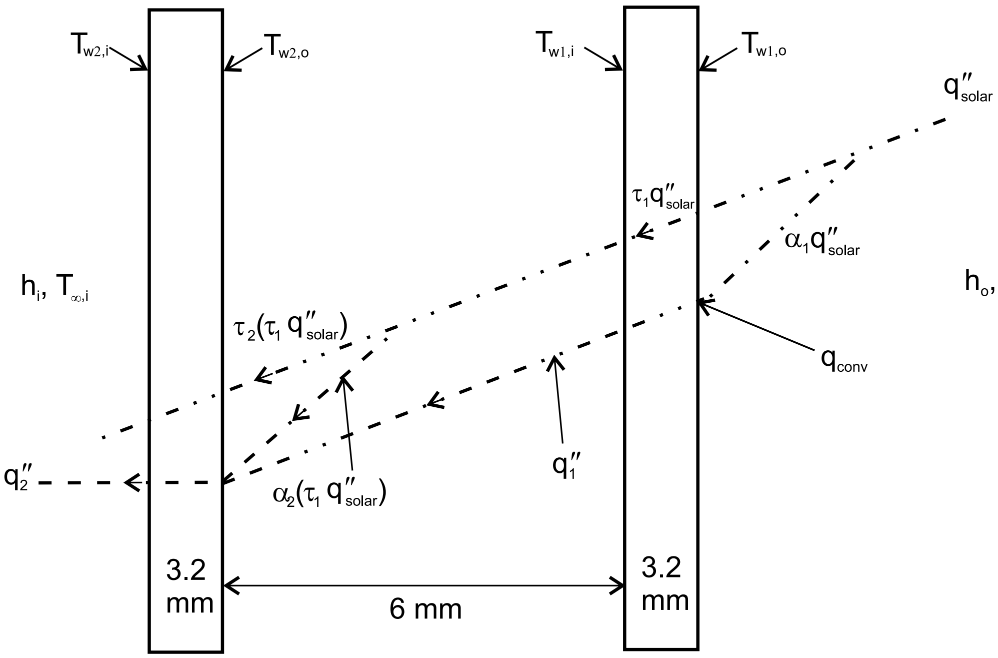

Figure 12.1

Given:

Thermal conductivity and thickness of the window glass, $k_g = 0.92$ W/mK, $L_g = 0.0032$ m

Thermal conductivity and thickness of the air, $k_a = 0.026$ W/mK, $L_a = 0.006$ m

Heat transfer coefficient of inner window glass, $h_i = 7.69$ W/m^2K

Heat transfer coefficient of outer window glass, $h_o = 25$ W/m^2K

Indoor room temperature, $T_{\infty,i} = 27$ °C

Ambient temperature, $T_{\infty,o}$ = 39 °C

Incident solar radiation on the wall, q'_{solar} = 680 W/m^2

For 60° incidence angle and double-pane window,

Absorptivity of the outer glass window pane, $\alpha_1 = 0.12$

Absorptivity of the inner glass window pane, $\alpha_2 = 0.08$

Transmissivity of both the glasses, $\tau = \tau_1 = \tau_2 = 0.75$

Let the cross-sectional area of the window be A m^2

i) From the heat balance at the outer surface of the outer glass window:

$$q_{rad} + q_{conv} = q_{cond} = q_1^{"} \quad \textbf{...(12.1)}$$

$$\alpha_1 \times q_{solar}^{"} \times A + h_o\left(T_{\infty,o} - T_{w1,o}\right)A = \frac{T_{w1,o} - T_{w2,o}}{R_{cond,g} + R_{cond,a}} A$$

$$0.12 \times 680 + 25\left(39 - T_{w1,o}\right) = \frac{T_{w1,o} - T_{w2,o}}{\frac{0.0032}{0.92} + \frac{0.006}{0.026}}$$

$$81.6 + 975 - 25T_{w1,o} = 4.27\left(T_{w1,o} - T_{w2,o}\right) \quad \textbf{...(12.2)}$$

From the heat balance at the outer surface of the inner glass window pane:

$$q_{rad} + q_1^{"} = q_2^{"} \quad \textbf{...(12.3)}$$

$$\tau_1 \times \alpha_2 \times q_{solar}^{"} \times A + \frac{T_{w1,o} - T_{w2,o}}{R_{cond,g} + R_{cond,a}} A = \frac{T_{w2,o} - T_{\infty,i}}{R_{cond,g} + R_{conv,i}} A$$

$$0.08 \times 0.75 \times 680 + \frac{T_{w1,o} - T_{w2,o}}{\frac{0.0032}{0.92} + \frac{0.006}{0.026}} = \frac{T_{w2,o} - T_{\infty,i}}{\frac{0.0032}{0.92} + \frac{1}{7.69}}$$

$$40.8 + \frac{T_{w1,o} - T_{w2,o}}{0.2342} = \frac{T_{w2,o} - 27}{0.1335}$$

$$T_{w2,o} = 0.36 \times T_{w1,o} + 20.67 \quad \textbf{...(12.4)}$$

From equations (12.2) and (12.4),the following can be obtained:

$$T_{w1,o} = 41.28 \text{ °C and } T_{w2,o} = 35.53 \text{ °C} \quad \textbf{...(12.5)}$$

Heat flux conducted through the outer glass window,

$$q_1^{"} = q_{conv}^{"} + q_{rad}^{"} = 25\left(39 - 41.28\right) + 0.12 \times 680 = 24.6 \ W / m^2 \qquad \textbf{...(12.6)}$$

Temperature at the inner surface of the outer glass window,

$$q_1^{"} = \frac{T_{w1,o} - T_{w1,i}}{R_g} \qquad \textbf{...(12.7)}$$

$$24.6 = \frac{41.28 - T_{w1,i}}{\frac{0.0032}{0.92}}$$

$$T_{w1,i} = 41.19 \text{ °C}$$

Heat flux conducted through the inner glass window,

$$q_2^{"} = q_1^{"} + \tau \times \alpha_2 \times q_{solar}^{"} = 24.6 + 40.8 = 65.4 \ W / m^2 \qquad \textbf{...(12.8)}$$

Temperature at the inner surface of the inner glass window,

$$q_2^{"} = \frac{T_{w2,o} - T_{w2,i}}{R_g} \qquad \textbf{...(12.9)}$$

$$65.4 = \frac{35.53 - T_{w1,i}}{\frac{0.0032}{0.92}}$$

$$T_{w1,i} = 35.30 \text{ °C}$$

Total heat flux through the double-pane glass window,

$$q_{total}^{"} = \tau_1 \times \tau_2 \times q_{solar}^{"} + q_2^{"} = 447.9 \ W / m^2 \qquad \textbf{...(12.10)}$$

ii) Heat flux calculation using SHGC:

Heat flux,

$$q_{total}^{"} = U\left(T_{\infty,o} - T_{\infty,i}\right) + SHGC \times q_{solar}^{"} \qquad \textbf{...(12.11)}$$

where, U is the overall heat transfer coefficient of the glass and SHGC is the solar heat gain coefficient.

Here,

$$\frac{1}{U}=\frac{1}{h_i}+\frac{L_g}{k_g}+\frac{L_g}{k_g}+\frac{L_a}{k_a}+\frac{1}{h_o}=\frac{1}{7.69}+\frac{0.0032}{0.92}+\frac{0.006}{0.026}+\frac{0.0032}{0.92}+\frac{1}{25} \quad \text{...(12.12)}$$

$$U=2.45\ W/m^2K$$

SHGC:

$$SHGC=\tau_1\times\tau_2+\alpha_1\frac{U}{h_o}+\alpha_1\times U\left(\frac{L_a}{k_a}+\frac{1}{h_o}\right) \quad \text{...(12.13)}$$

$$SHGC=0.75\times0.75+0.12\frac{2.45}{25}+0.08\times2.45\left(\frac{0.006}{0.026}+\frac{1}{25}\right)=0.63$$

Therefore,

$$q''_{total}=2.45(39-27)+0.63\times680=457.8\ W/m^2 \quad \text{...(12.14)}$$

REMARKS

It can be observed that the double-pane window can reduce the heat transfer significantly (~25% reduction) compared to the single-pane window as mentioned in the previous problem. It is also observed that the difference of heat transfer calculation between the studies considering energy balance and following the concept of SHGC is very less (<10 W/m^2).

PROBLEM 13

Define sol-air temperature. Find out the sol-air temperature for Raipur in the month of May between 8.00 a.m. and 6.00 p.m. at an interval of two hours. Monthly average incident solar radiation at different walls/ roof and ambient air temperature for a typical building located in Raipur are as follows:

Table 13.1

	I_{solar} (W/m^2)					T_∞ (°C)
Time	**East wall**	**West wall**	**North wall**	**South wall**	**Roof**	**Outdoor air temperature**
8.00 a.m.	642.9	123.8	213.9	123.9	440.2	27.2
10.00 a.m.	575.9	163.3	180.2	168.6	779.7	30.9
12.00 Noon	205.1	152.3	152.1	189.9	914.9	35.5
2.00 p.m.	155.8	472.0	164.6	170.0	784.2	38.6
4.00 p.m.	112.2	529.1	169.9	112.2	423.9	39.4
6.00 p.m.	18.9	115.2	46.5	18.9	44.7	37.6

Consider convective and radiative heat transfer coefficients associated with the outside surface of the walls/ roof as 15 W/m^{2}K and 10 W/m^{2}K, respectively. Absorptivity of the wall and roof can be taken as 0.8.

SOLUTION

Sol-air temperature: Sol-air temperature is an equivalent outside air temperature defined in such a way that the total heat transferred from the environment to the outside surface of the wall is same as the combined effect of the actual temperature gradient between the outside air temperature and the outside surface of the wall, and the incident solar radiation. The sol-air temperature can be used instead of outside air temperature and the incident solar radiation separately.

Consider case (a) shown in Figure13.1 with convective and radiative heat transfer coefficients as h_c and h_r, respectively. Absorptivity and total incident solar radiation at the outer surface of the wall are α and I_{solar}, respectively. The outside air temperature and the temperature of the outer surface of the wall are T_∞ and T_{surf}, respectively.

The rate of heat transfer from the environment to the outside surface of the wall:

$$q'' = (h_c + h_r)(T_\infty - T_{surf}) + \alpha I_{solar} \quad \text{...(13.1)}$$

Introducing an equivalent temperature (i.e., sol-air temperature), the rate of heat transfer from the environment to the outside surface of the wall can be written as:

$$q'' = (h_c + h_r)(T_{sol\text{-}air} - T_{surf}) \quad \text{...(13.2)}$$

From Eq. (13.1) and Eq. (13.2), the sol-air temperature can be obtained as:

$$T_{sol-air} = T_\infty + \frac{\alpha I_{solar}}{(h_c + h_r)} \quad \text{...(13.3)}$$

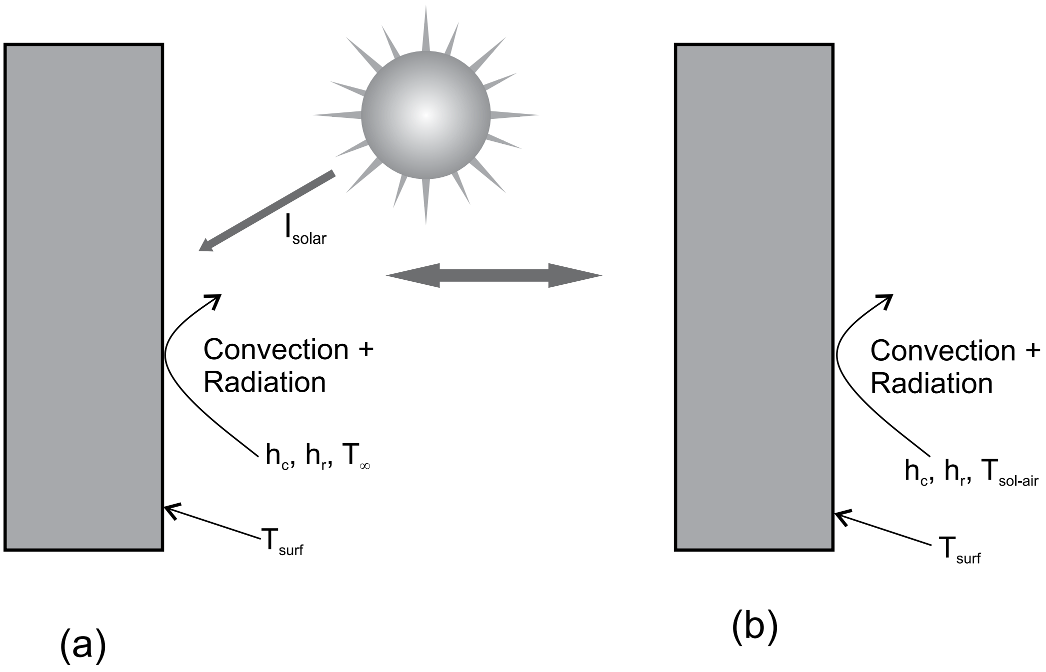

Figure 13.1

Given:

Convective heat transfer coefficient, h_c= 15 W/m^{2}K

Radiative heat transfer coefficient, h_r = 10 W/m^{2}K

Absorptivity of the wall/roof surfaces, $\alpha = 0.8$

Ambient air temperature, T_∞ = Refer table 13.1

Incident solar radiation, I_{solar} = Refer table 13.1

The sol-air temperature for various walls/roof at different time of the day can be calculated using Eq. (13.3). The obtained sol-air temperatures are as given in Table 13.2.

Table 13.2

Sol-air temperature, $T_{sol\text{-}air}$ (°C)					
Time	**East wall**	**West wall**	**North wall**	**South wall**	**Roof**
8.00 a.m.	47.77	31.16	34.05	31.17	41.29
10.00 a.m.	49.33	36.13	36.67	36.30	55.85
12.00 Noon	42.06	40.37	40.37	41.58	64.78
2.00 p.m.	43.59	53.70	43.87	44.04	63.69
4.00 p.m.	42.99	56.33	44.84	42.99	52.97
6.00 p.m.	38.21	41.29	39.09	38.21	39.03

REMARKS

Variations of the sol-air temperature obtained based on monthly average values of outside air temperature and the incident solar radiation are shown in Figure13.2 for different walls and roof of a typical building located in Raipur. It can be observed that the sol-air temperatures corresponding to various building walls/roof are significantly higher than the ambient air temperature. The sol-air temperature is found highest for roof with peak at noon followed by the west wall in the afternoon. This effectively means that the maximum heat will enter the building through the roof, followed by the west wall. The sol-air temperature corresponding to the east wall decreases with time primarily due to decrease in the incident solar radiation with time of the day. On the other hand, the sol-air temperature corresponding to the west wall increases significantly during the second half of the day due to increase in the incident solar radiation in the afternoon.

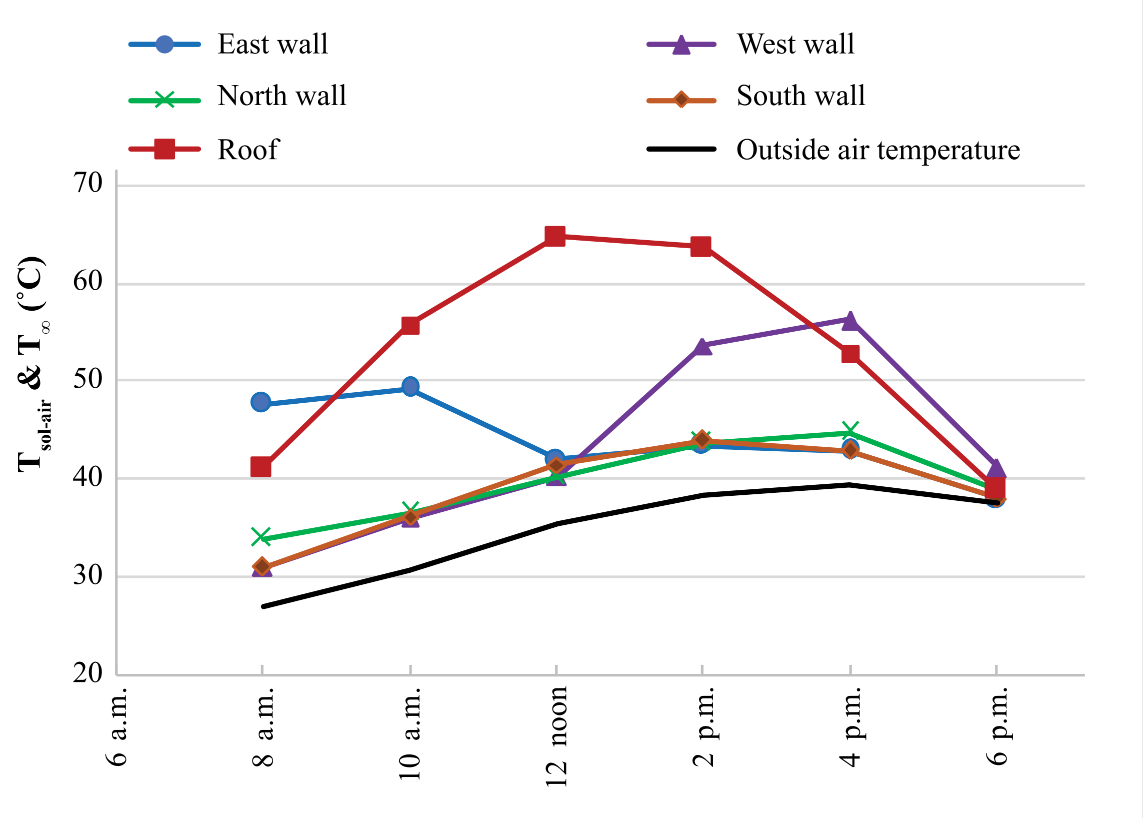

Figure 13.2

PROBLEM 14

Consider a room (maintained at 27 °C to achieve thermal comfort) in a building located in Jaipur with exposed west wall of 5 × 3.5 m. On a typical day in May at 4.00 p.m., the incident solar radiation on the west wall is 680 W/m^2 and the ambient temperature is 39 ºC. The wall consists of 230-mm thick brick layer with a plaster layer of 15-mm thickness on both sides of the wall. A single-pane glass window of 1 × 1 m^2 (having thickness 3.2 mm) is also provided in the west wall.

- Determine the overall heat transfer rate through the wall incorporating window considering steady state condition. Repeat the calculation for a case when there is no window and compare the results.
- Also vary the area of the window and show variation of the overall heat transfer rate with window-to-wall ratio (WWR) and discuss the results.

Thermal conductivity, transmittivity, and absorptivity of the glass (for 60° incidence angle) are 0.92 W/mK, 0.75, and 0.11, respectively. The average heat transfer coefficients associated with the inner and outer surfaces of the wall/window are 7.69 W/m^{2}K and 25.0 W/m^{2}K, respectively. Thermal conductivities of brick and plaster are 0.85 W/mK and 0.72 W/mK, respectively. Take absorptivity of the exposed surface of the plaster as 0.65.

SOLUTION

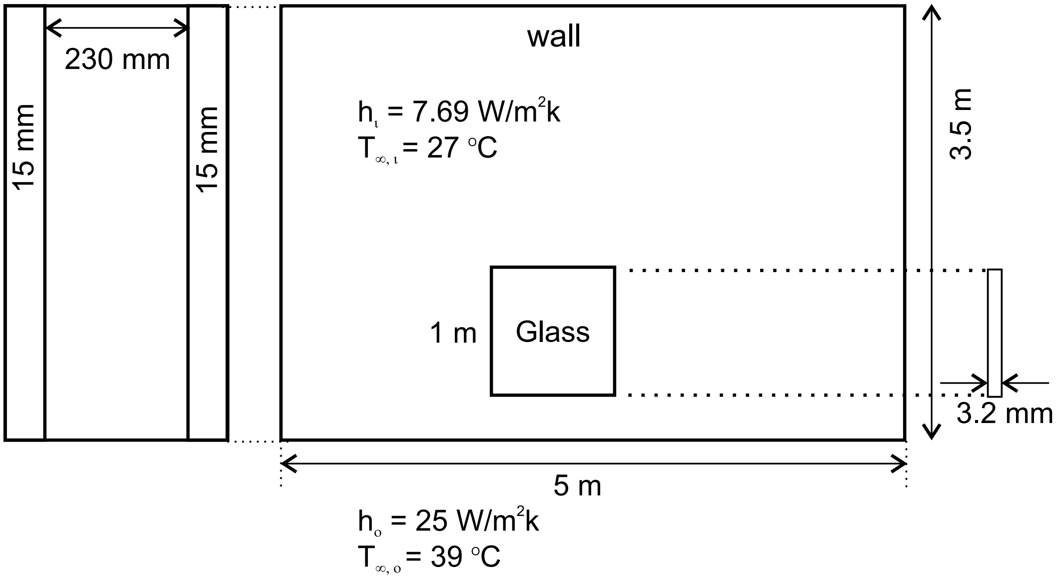

Figure 14.1

Given:

Heat transfer coefficient of inner wall/window glass, h_i = 7.69 W/m^{2}K

Heat transfer coefficient of outer wall/window glass, h_o = 25 W/m^{2}K

Indoor room temperature, $T_{\infty,i}$ = 27 °C

Ambient temperature, $T_{\infty,o}$ = 39 °C

Incident solar radiation on the wall, q''_{solar} = 680 W/m^2

Heat transfer through the single-pane glass window:

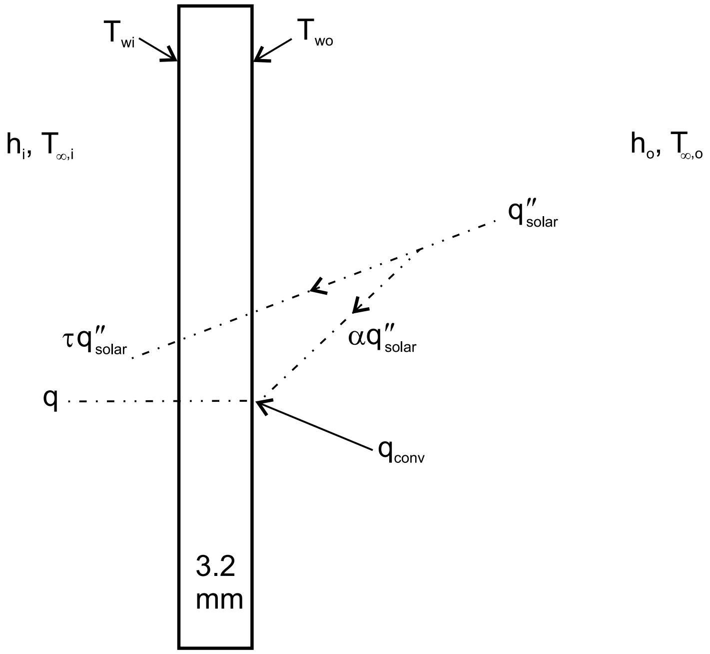

Figure 14.2

Thermal conductivity and thickness of the window glass, $k_g = 0.92$ W/mK, $L_g = 0.0032$ m

For 60° incidence angle,

Absorptivity of the glass, $\alpha = 0.11$

Transmissivity of the glass, $\tau = 0.75$

Let cross-sectional area of the window be A_g m^2.

From the heat balance at the outer surface of the window:

$$q_{rad} + q_{conv} = q_{cond} = q \qquad \textbf{...(14.1)}$$

$$\alpha \times q''_{solar} \times A_g + h_o\left(T_{\infty,o} - T_{w,o}\right)A_g = \frac{T_{w,o} - T_{\infty,i}}{R_{cond} + R_{conv,in}} A_g$$

$$0.11 \times 680 A_g + 25\left(39 - T_{w,o}\right)A_g = \frac{T_{w,o} - 27}{\dfrac{0.0032}{0.92} + \dfrac{1}{7.69}} A_g$$

$$T_{w,o} = 38.54 \text{ °C}$$

Heat flux,

Heat conducted through the window,

$$q'' = \frac{38.54-27}{\frac{0.1335}{A}A_g} = 86.44\ W/m^2 \qquad \textbf{...(14.2)}$$

Heat transmitted through the window,

$$q''_{trans} = \tau \times q''_{solar} = 510\ W/m^2 \qquad \textbf{...(14.3)}$$

Total heat transfer,

$$q''_{total} = q'' + q''_{trans} = 86.44 + 510 = 596.44\frac{W}{m^2} = q''_g \qquad \textbf{...(14.4)}$$

Heat transfer through the brick wall:

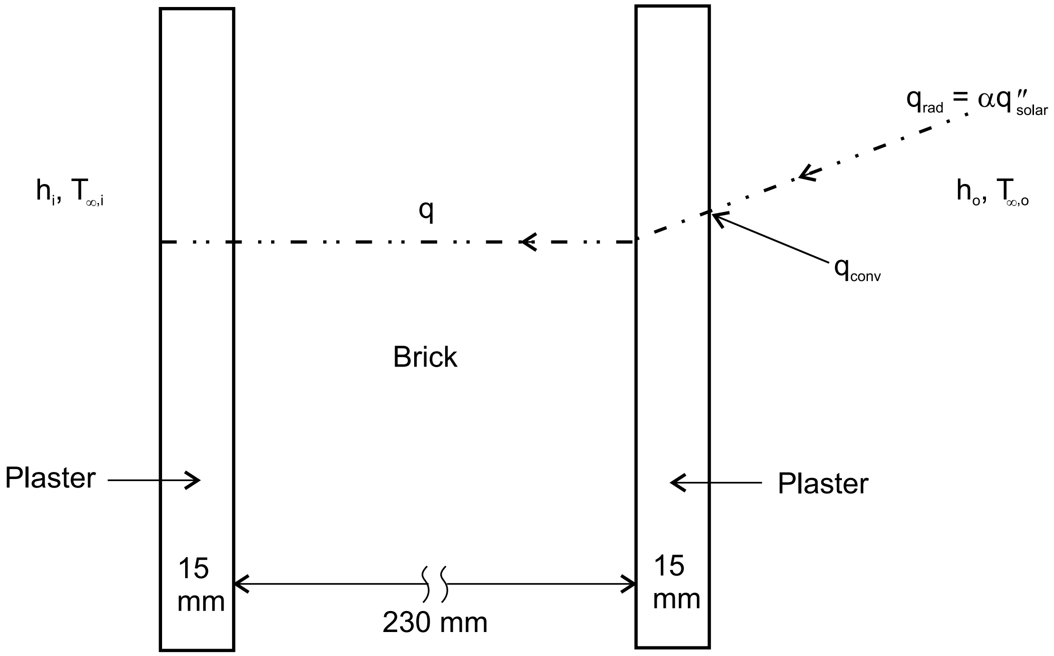

Figure 14.3

Thermal conductivity and thickness of plaster, $k_p = 0.72$ W/mK, $L_p = 0.015$ m

Thermal conductivity and thickness of brick, $k_b = 0.85$ W/mK, $L_b = 0.115$ m

Absorptivity of the wall, $\alpha_w = 0.65$

Let cross-sectional area of the wall be A m^2.

Conductive thermal resistance,

$$R_{cond} = \frac{L_p}{k_p A} + \frac{L_b}{k_b A} + \frac{L_p}{k_p A} \qquad \textbf{...(14.5)}$$

$$R_{cond} = \frac{0.015}{0.72A} + \frac{0.23}{0.85A} + \frac{0.015}{0.72A}$$

$$R_{cond} = \frac{0.31}{A}\frac{K}{W}$$

Convective thermal resistance:

Inner wall convective thermal resistance,

$$R_{conv,in} = \frac{1}{h_i A} = \frac{1}{7.69A} = \frac{0.13}{A}\frac{K}{W} \qquad \textbf{...(14.6)}$$

Outer wall convective thermal resistance,

$$R_{conv,out} = \frac{1}{h_0 A} = \frac{1}{25A} = \frac{0.04}{A}\frac{K}{W} \qquad \textbf{...(14.7)}$$

From the heat balance at the outer surface of the west wall for steady state condition:

$$q_{rad} + q_{conv} = q_{rad} = q \qquad \textbf{...(14.8)}$$

$$680 \times 0.65 \times A + \frac{39 - T_{out,w}}{0.04/A} = \frac{T_{out,w} - 27}{(0.31 + 0.13)/A}$$

$$442 + 975 - 25\ T_{out,w} = 2.27\ T_{out,w} - 61.36$$

$$T_{out,w} = 54.21\ °\mathrm{C}$$

Heat transfer rate,

$$q = \frac{T_{out,w} - \mathrm{T}_{\infty,\mathrm{i}}}{R_{cond} + R_{conv,in}} = \frac{54.21 - 27}{0.31 + 0.13} A = 61.84A\ W \qquad \textbf{...(14.9)}$$

Heat flux,

$$q'' = \frac{q}{A} = \frac{61.84}{A} = 61.84\ W/m^2 = q''_{bw} \qquad \textbf{...(14.10)}$$

Window-to-wall ratio (WWR):

$$WWR = \frac{Area\ of\ the\ window}{Total\ area\ of\ the\ wall} = \frac{1\times 1}{5\times 3.5} = 5.71\% \qquad \text{...(14.11)}$$

Overall heat transfer rate,

$$q_{total} = q_g'' \times A_g + q_{bw}'' \times A = 596.44\times 1 + 61.84\times 16.5 = 1616.80\ W \qquad \text{...(14.12)}$$

Whenthere is no window, then WWR = 0%

Overall heat transfer rate,

$$q_{total} = q_g'' \times A_g + q_{bw}'' \times A = 596.44\times 0 + 61.84\times 17.5 = 1082.20\ W \qquad \text{...(14.13)}$$

Similarly, when the size of the window is 1.5 × 1.5 m^2,

$$WWR = \frac{1.5\times 1.5}{5\times 3.5} = 12.86\% \qquad \text{...(14.14)}$$

Overall heat transfer rate,

$$q_{total} = q_g'' \times A_g + q_{bw}'' \times A = 596.44\times 2.25 + 61.84\times 15.25 = 2285.05\ W \qquad \text{...(14.15)}$$

When the size of the window is 2 × 2 m^2,

$$WWR = \frac{2\times 2}{5\times 3.5} = 22.86\% \qquad \text{...(14.16)}$$

Overall heat transfer rate,

$$q_{total} = q_g'' \times A_g + q_{bw}'' \times A = 596.44\times 4 + 61.84\times 13.5 = 3220.60\ W \qquad \text{...(14.17)}$$

When the size of the window is 3 × 3 m^2,

$$WWR = \frac{3\times 3}{5\times 3.5} = 51.43\% \qquad \text{...(14.18)}$$

Overall heat transfer rate,

$$q_{total} = q_g'' \times A_g + q_{bw}'' \times A = 596.44\times 9 + 61.84\times 8.5 = 5893.60\ W \qquad \text{...(14.19)}$$

When the size of the window is 4 × 3.5 m^2,

$$WWR = \frac{4\times 3.5}{5\times 3.5} = 80\% \qquad \text{...(14.20)}$$

Overall heat transfer rate,

$$q_{total} = q_g'' \times A_g + q_{bw}'' \times A = 596.44\times 14 + 61.84\times 3.5 = 8566.60\ W \qquad \text{...(14.21)}$$

When the size of the glazing is 5 × 3.5 m^2,

$$WWR = \frac{5 \times 3.5}{5 \times 3.5} = 100\% \qquad \text{...(14.22)}$$

Overall heat transfer rate,

$$q_{total} = q''_g \times A_g + q''_{bw} \times A = 596.44 \times 17.5 + 61.84 \times 0 = 10437.70\ W \qquad \text{...(14.23)}$$

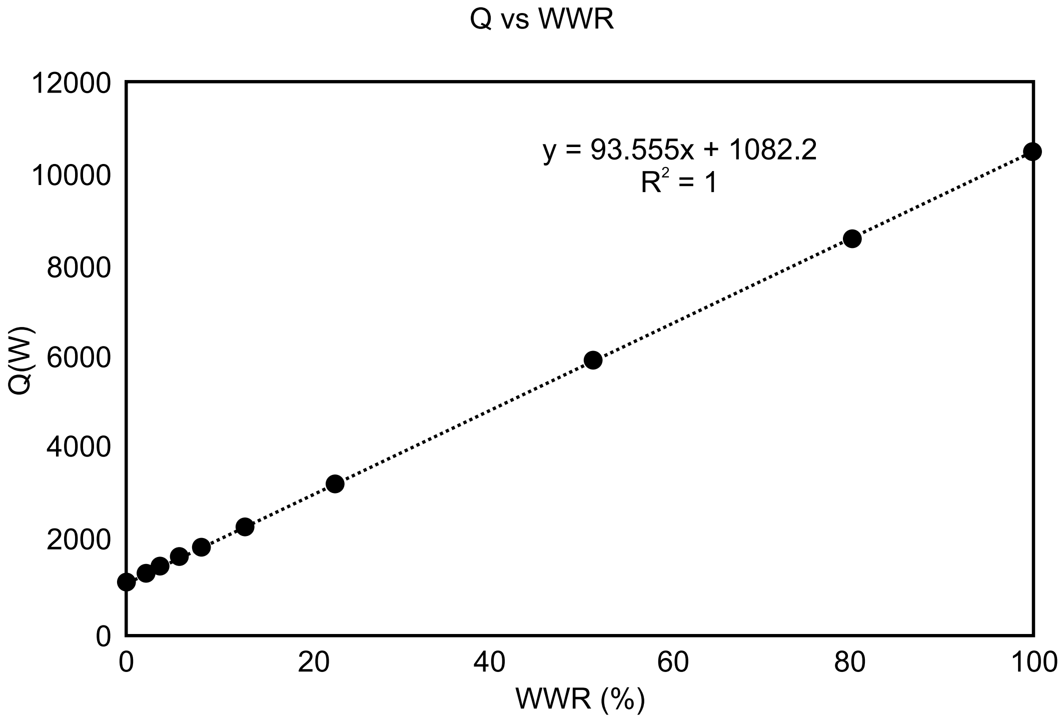

Figure 14.4

REMARKS

It can be observed that with the introduction of window (WWR ratio = 5.71%), the heat transfer through the external wall increases significantly (≈50%). It is also observed that the heat transfer rate increases linearly with increase in the WWR. Hence, an optimum WWR needs to be considered while designing a building, considering the building cooling load, natural ventilation, and day lighting aspects. WWR of 15%–30% is sufficient for Indian conditions to meet the requirements of natural ventilation and day lighting. The optimum WWR, however, also depends on other factors such as orientation.

PROBLEM 15

Consider a room in a building located at Raipur, consisting of a window in the west wall. The average incident solar radiation in the month of May at 4.00 p.m. on the exposed west wall is 529 W/m^2 and the outdoor temperature is 39 °C. The room is maintained at 27 °C to achieve thermal comfort. The average heat transfer coefficients associated with the inner and outer surfaces of the window are 7.69 W/m^{2}K and 25 W/m^{2}K, respectively. Consider two types of window configurations:

- ***Single-pane window with external shading:*** The window has single-pane glass of thickness 5.7 mm. External shading, made up of textile material (Suntex® 95 black), is placed at a distance of 42 mm from the outer surface of the glass window. The thickness of the shading layer is 0.7 mm.
- ***Double-pane window without shading:*** The window has two glass panes with air gap in-between. The thickness of each glass pane is 5.7 mm and the air gap between the glass panes is 12 mm.

Find out the heat transfer to the room through the window (in W/m^2) and the temperature of the inner-most surface of the glass for both the cases considering steady state condition. Discuss the results and describe which window configuration is better.

Given: Thermal conductivities of glass, air, and Suntex® 95 black are 1 W/mK, 0.026 W/mK, and 0.2 W/mK, respectively. Absorptivities of the glass and Suntex® 95 black are 0.16 and 0.9, respectively. Transmittivities of the glass and Suntex® 95 black are 0.77 and 0.07, respectively.

Hint: Neglect transmission through the external shading layer as transmissivity of the material is very less. Radiation heat transfer between the outer surface of the glass and the inner surface of the external shading layer can also be neglected for simplification. Also, neglect the air movement between the air gap of external shading layer and the window glass.

SOLUTION

1. Single-pane window with external shading:

Given,

Thermal conductivity and thickness of the window glass, $k_g = 1$ W/mK, $L_g = 0.0057$ m

Thermal conductivity and thickness of the air layer, $k_a = 0.026$ W/mK, $L_a = 0.042$ m

Thermal conductivity and thickness of the Suntex® 95 black, $k_{st} = 0.2$ W/mK, $L_{st} = 0.0007$ m

Average heat transfer coefficient of the inner surface of the window, $h_i = 7.69$ W/m^{2}K

Average heat transfer coefficient of the outer surface of the window/shading, $h_o = 25$ W/m^{2}K

Indoor room temperature, $T_{\infty,i} = 27$ °C

Outdoor ambient temperature, $T_{\infty,o} = 39$ °C

Incident solar radiation on the wall, $q''_{solar} = 529$ W/m^2

Absorptivity of the Suntex® 95 black, $\alpha_{st} = 0.11$

Assumptions:

a) Neglect solar transmission through the Suntex® 95 black shading

b) Neglect the radiation heat exchange between glass and shading

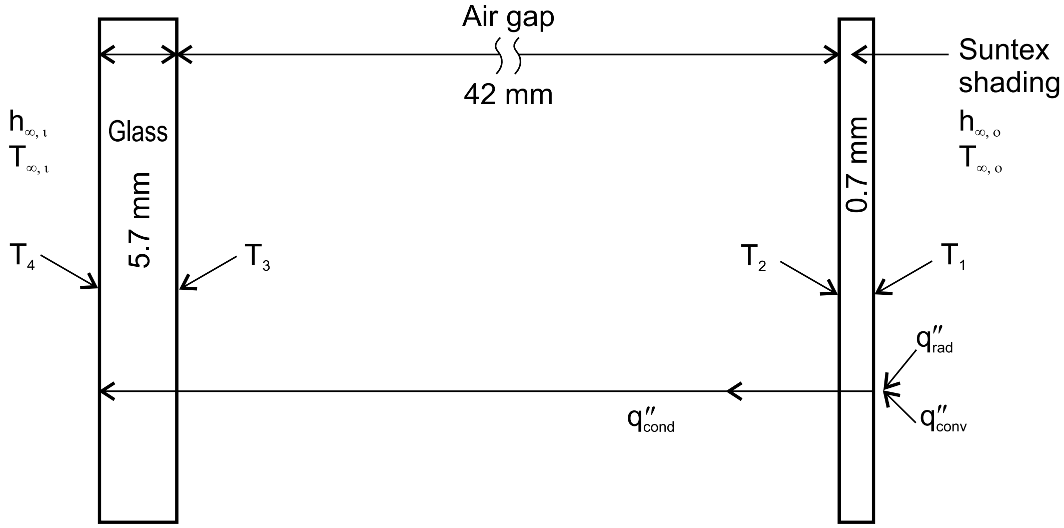

Figure 15.1

From the heat balance at the outer surface of the Suntex® 95 black shading:

T_1:

$$q''_{rad} + q''_{conv} = q''_{cond} \qquad \textbf{...(15.1)}$$

$$\alpha \times q''_{solar} + h_o\left(T_{\infty,o} - T_1\right) = \frac{T_1 - T_{\infty,i}}{R_{cond} + R_{conv,in}}$$

$$0.9 \times 529 + 25\left(39 - T_1\right) = \frac{T_1 - 27}{\frac{0.0007}{0.2} + \frac{0.042}{0.026} + \frac{0.0057}{1} + \frac{1}{7.69}}$$

$$T_1 = 57.35\ °\text{C}$$

Total heat flux to the room:

$$q''_{cond} = q''_{room} = \frac{57.35 - 27}{1.75} = 17.34\ W/m^2 \qquad \textbf{...(15.2)}$$

T_2:

$$\frac{T_1 - T_2}{0.0007/0.2} = 17.34 \Rightarrow T_2 = 57.29\ °\text{C} \qquad \textbf{...(15.3)}$$

T_3:

$$T_3 = T_2 - q''_{cond} \times \frac{L_a}{k_a} \qquad \text{...(15.4)}$$

$$T_3 = 57.29 - 17.34 \times \frac{0.042}{0.026} \Rightarrow T_3 = 29.28 \text{ °C}$$

T_4:

$$T_4 = 29.28 - 17.34 \times \frac{0.0057}{1} \Rightarrow T_4 = 29.18 \text{ °C} \qquad \text{...(15.5)}$$

2. Double-pane window without shading:

Given,

Thermal conductivity and thickness of the window glass, k_g = 1 W/mK, L_g = 0.0057 m

Thermal conductivity and thickness of the air layer, k_a = 0.026 W/mK, L_a = 0.042 m

Absorptivity of the glass, α_g = 0.16

Transmittivity of the glass, τ_g = 0.77

All other indoor and outdoor conditions are the same as in the previous case.

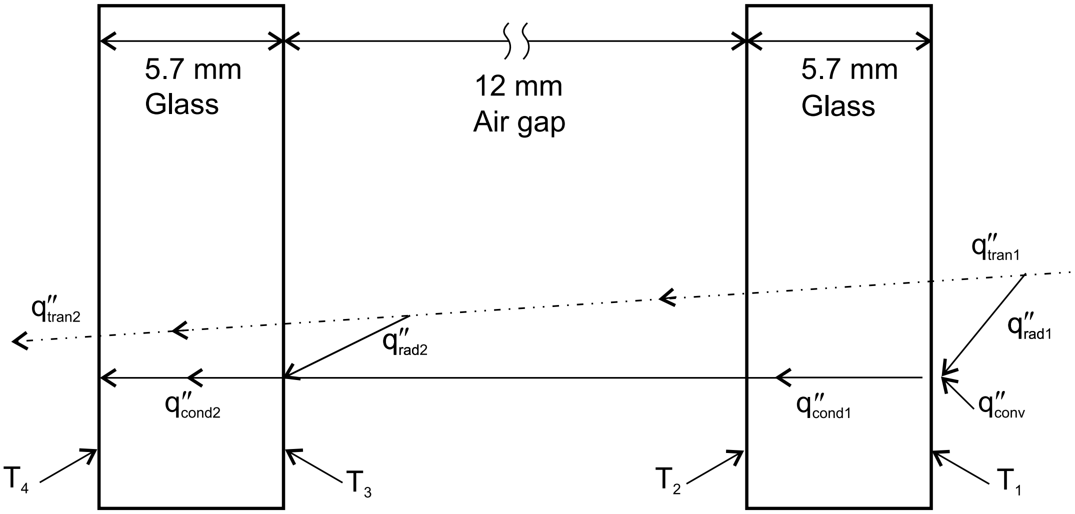

Figure 15.2

From the heat balance at the outer surface of the outer window glass:

$$q''_{cond1} = q''_{conv} + q''_{rad1}$$ **...(15.6)**

$$\frac{T_1 - T_3}{R_{cond}} = h_o\left(T_{\infty,o} - T_1\right) + \alpha \times q''_{solar}$$

$$\frac{T_1 - T_3}{\frac{0.0057}{1} + \frac{0.012}{0.026}} = 25\left(39 - T_1\right) + 0.16 \times 529$$

$$12.68\ T_1 - T_3 = 495.16$$ **...(15.7)**

From the heat balance at the outer surface of the inner window glass:

$$q''_{cond2} = q''_{cond1} + q''_{rad2}$$ **...(15.8)**

$$\frac{T_3 - T_{\infty,i}}{R_{cond2} + R_{conv,in}} = \frac{T_1 - T_3}{R_{cond1}} + \tau \times \alpha \times q''_{solar}$$

$$\frac{T_3 - 27}{\frac{0.0057}{1} + \frac{1}{7.69}} = \frac{T_1 - T_3}{\frac{0.0057}{1} + \frac{0.012}{0.026}} + 0.77 \times 0.16 \times 529$$

$$9.51\ T_3 - 2.14\ T_1 = 264.08$$ **...(15.9)**

Solving equations (15.7) and (15.9), the following can be obtained

$$T_1 = 41.98\ °\text{C and } T_3 = 37.20\ °\text{C}$$ **...(15.10)**

Heat conducted to the room:

$$q''_{cond2} = q''_{cond1} + q''_{rad2}$$ **...(15.11)**

$$q''_{cond2} = 2.14\left(T_1 - T_3\right) + 65.17 = 2.14\left(41.98 - 37.20\right) + 65.17 = 75.40 \frac{W}{m^2}$$

T_4:

$$\frac{T_3 - T_4}{R_{cond}} = q''_{cond2} = 75.40$$ **...(15.12)**

$$T_4 = 36.77\ °\text{C}$$

Total heat flux to the room:

$$q''_{room} = q''_{cond2} + q''_{tran2}$$ **...(15.13)**

$$q''_{room} = 75.40 + 0.77 \times 0.77 \times 529 = 389.04\ W / m^2$$

REMARKS

It is observed that a single-pane window with external shading is a much better option compared to a double-pane window in term of heat transfer to the room (cooling load) and inner-most layer surface temperature of window glass. This is because there is very small/almost no direct solar radiation falling on the window glass in the case of external shading. Please note that in the case of external shading, direct transmission from the shading layer is neglected; in real life, the heat transfer to the room and inner-most glass layer surface temperature will be more.

To understand a more realistic scenario, the heat flux from the single-pane window with external shading is also calculated using the SHGC[2] value given by the manufacturer. For the shading layer of Suntex® 95 black with 5.7-mm-thick single clear glass, the SHGC value is given as 0.16. The heat flux based on SHGC calculation is 91.36 W/m^2. It can be observed that the heat flux obtained from the SHGC calculation deviates significantly with the calculation shown above (17.34 W/m^2 compared to 91.36 W/m^2). This deviation is mainly because of simplifying assumptions considered in the above calculation of single-pane window with external shading. These assumptions were: (1) neglecting the transmission of solar radiation through the shading layer, and (2) neglecting the radiation heat exchange between the inner surface of shading layer and the outer surface of the glass. Taking into account the transmission of solar radiation through the shading layer (assumption 1), the heat flux is calculated as 51.50 W/m^2, which is a better estimation of the heat flux and it is closer to the value calculated based on SHGC. Results can be improved further by considering radiation heat exchange between the inner surface of the shading layer and the outer surface of the glass (assumption 2).

Calculations were also performed on just a single-pane glass window and a double-pane glass window with external shading (neglecting transmission of solar radiation and radiation exchange). A comparative study (in terms of room heat transfer and inner surface temperature) is done considering all the five window configurations and the results are shown in the table below:

Table 15.1

S. No.	Cases studied	Room heat transfer (W/m^2)	Innermost surface temperature (°C)
1	Single-pane glass window	494.87	38.38
2	Double-pane glass window	389.04	36.77
3	Single-pane glass window with external shading (neglecting solar transmission through shading and radiation exchange between glass and shading)	17.34	29.18
4	Double-pane glass window with external shading (neglecting solar transmission through shading and radiation exchange between glass and shading)	13.74	28.77
5	Single-pane glass window with external shading (considering solar transmission through shading but neglecting radiation exchange between glass and shading)	51.50	29.99
6	Single-pane glass window with external shading (based on SHGC calculation)	91.36	-

---

[2] SHGC or Solar Heat Gain Coefficient is the fraction of incident solar radiationadmitted through non-opaque components, both directly transmitted, and absorbedand subsequently released inward through conduction, convection, and radiation.

The following conclusions can be drawn based on the above study:

Single-pane window with external shading is a better option compared to a double-pane window as far as heat transfer or thermal comfort or building energy efficiency is considered.

Although the heat transfer through windows can be reduced further by employing a double-pane window with external shading, the reduction is not significant as compared to a single-pane window with external shading. So, considering cost and maintenance, a single-pane window with external shading can be preferred.

Supplementary calculations:

 # Single-pane glass window:

All the properties, indoor, and outdoor conditions are the same as in the case of double-pane windows.

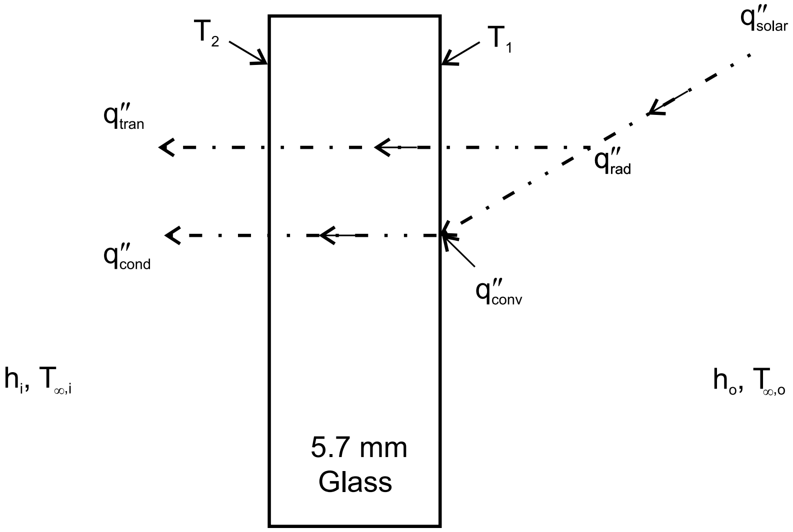

Figure 15.3

From the heat balance at the outer surface of the outer window glass:

$$q''_{cond} = q''_{conv} + q''_{rad} \qquad \textbf{...(15.14)}$$

$$\frac{T_1 - T_{\infty,i}}{R_{cond} + R_{conv,in}} = h_o\left(T_{\infty,o} - T_1\right) + \alpha \times q''_{solar}$$

$$\frac{T_1 - 27}{\dfrac{0.0057}{1} + \dfrac{1}{7.69}} = 25\left(39 - T_1\right) + 0.16 \times 529$$

$$T_1 = 38.88 \text{ °C}$$

Heat conducted to the room:

$$q''_{cond} = \frac{T_1 - T_{\infty,i}}{R_{cond} + R_{conv,in}} = \frac{38.88 - 27}{0.1357} = 87.54\ W/m^2 \qquad \text{...(15.15)}$$

Heat transmitted to the room:

$$q''_{tran} = \tau \times q''_{solar} = 0.77 \times 529 = 407.33\ W/m^2 \qquad \text{...(15.16)}$$

Total heat flux to the room:

$$q''_{room} = q''_{cond} + q''_{tran} \qquad \text{...(15.17)}$$

$$q''_{room} = 87.54 + 407.33 = 494.87\ W/m^2$$

T_2:

$$T_2 = T_1 - q''_{cond} \times R_{cond} = 38.38\ °\text{C} \qquad \text{...(15.18)}$$

 # Single-pane glass window with external shading (considering solar transmission through shading but neglecting radiation exchange between glass and shading)

All the properties, indoor and outdoor conditions are the same as in the previous cases (also shown in the figure) except one added property, transmittivity of the Suntex® 95 black, $\tau_{st} = 0.07$ is considered.

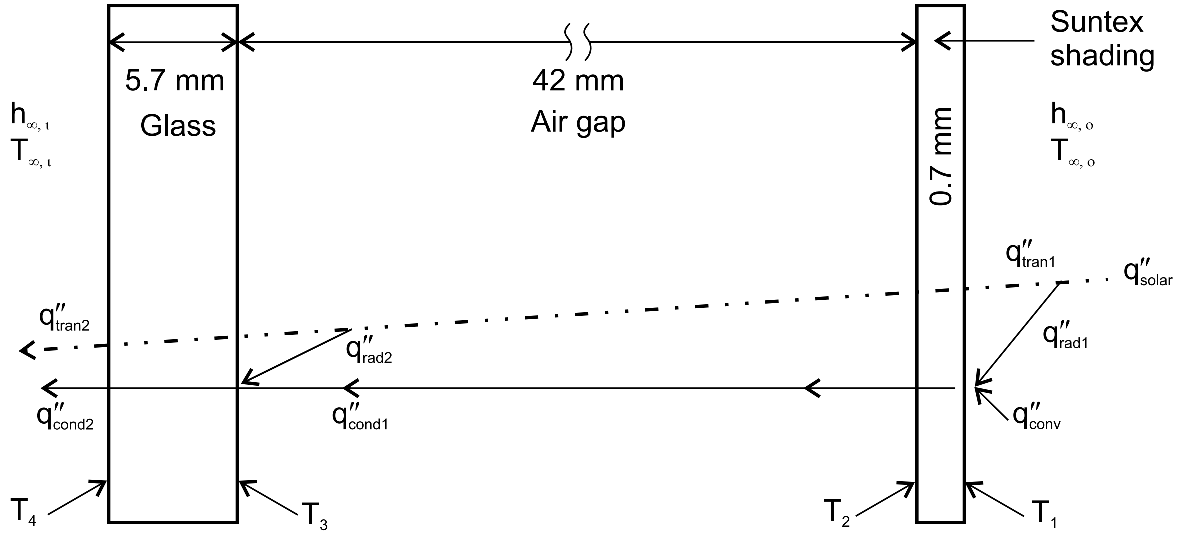

Figure 15.4

From the heat balance at the outer surface of the Suntex® 95 black shading:

$$q''_{cond1} = q''_{conv} + q''_{rad1} \qquad \text{...(15.19)}$$

$$\frac{T_1 - T_3}{R_{cond1}} = h_o\left(T_{\infty,o} - T_1\right) + \alpha_{st} \times q''_{solar}$$

$$\frac{T_1 - T_3}{\frac{0.0007}{0.2} + \frac{0.042}{0.026}} = 25\left(39 - T_1\right) + 0.9 \times 529$$

$$41.5T_1 - T_3 = 2350.78 \qquad \text{...(15.20)}$$

`From the heat balance at the outer surface of the window glass:

$$q''_{cond2} = q''_{cond1} + q''_{rad2} \qquad \text{...(15.21)}$$

$$\frac{T_3 - T_{\infty,i}}{R_{cond2} + R_{conv,in}} = \frac{T_1 - T_3}{R_{cond1}} + \tau_{st} \times \alpha_g \times q''_{solar}$$

$$\frac{T_3 - 27}{\frac{0.0057}{1} + \frac{1}{7.69}} = \frac{T_1 - T_3}{\frac{0.0007}{0.2} + \frac{0.042}{0.026}} + 0.07 \times 0.16 \times 529$$

$$0.62\ T_3 - 7.99\ T_1 = -204.83 \qquad \text{...(15.22)}$$

Solving equations (15.20) and (15.22), the following can be obtained:

$$T_1 = 57.37\ \text{°C and } T_3 = 30.12\ \text{°C} \qquad \text{...(15.23)}$$

Heat conducted to the room:

$$q''_{cond2} = q''_{cond1} + q''_{rad2} \qquad \text{...(15.24)}$$

$$q''_{cond2} = \frac{30.12 - 27}{0.1357} = 22.99 \frac{W}{m^2}$$

Heat transmitted to the room:

$$q''_{tran2} = \tau \times q''_{solar} = 0.07 \times 0.77 \times 529 = 28.51\ W/m^2 \qquad \text{...(15.25)}$$

Total heat flux to the room:

$$q''_{room} = q''_{cond2} + q''_{tran2} \qquad \text{...(15.26)}$$

$$q''_{room} = 22.99 + 28.51 = 51.50\ W/m^2$$

T_4:

$$\frac{T_3 - T_4}{R_{cond,g}} = q''_{cond2} = 22.99 \qquad \text{...(15.27)}$$

$$T_4 = 29.99\ °C$$

***# Double-pane glass window with external shading (neglecting solar transmission through shading and radiation exchange between glass and shading)*:**

All the properties, indoor and outdoor conditions are the same as in the previous cases.

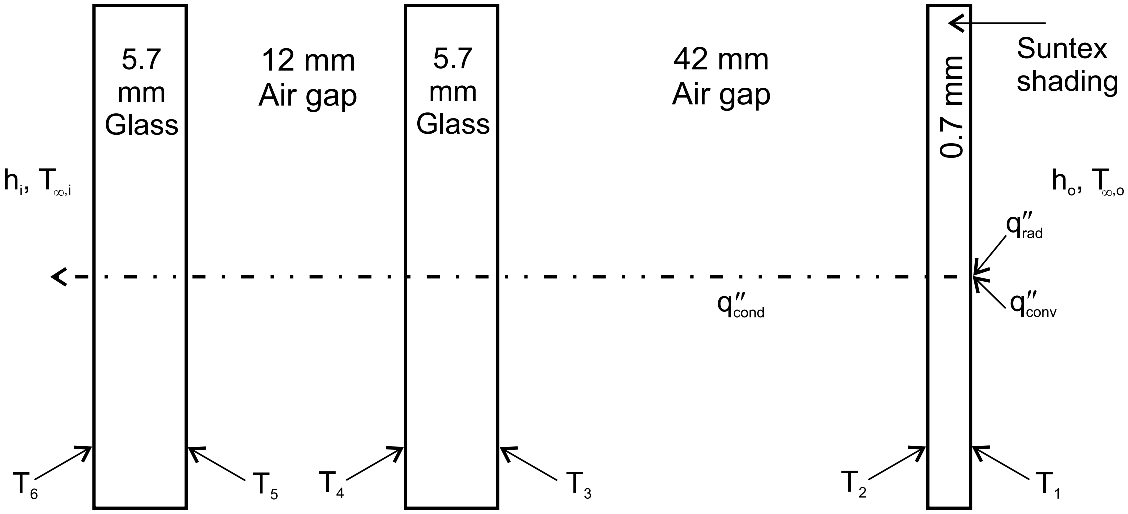

Figure 15.5

Conduction resistance,

$$R_{cond} = 2\frac{0.0057}{1} + \frac{0.012}{0.026} + \frac{0.042}{0.026} + \frac{0.0007}{0.2} = 2.09\frac{Km^2}{W}$$

From the heat balance at the outer surface of the Suntex® 95 black shading:

$$q''_{cond} = q''_{conv} + q''_{rad} \qquad \text{...(15.28)}$$

$$\frac{T_1 - T_{i,\infty}}{R_{cond} + R_{conv,in}} = h_o\left(T_{\infty,o} - T_1\right) + \alpha_{st} \times q''_{solar}$$

$$\frac{T_1 - 27}{2.09 + \dfrac{1}{7.69}} = 25\left(39 - T_1\right) + 0.9 \times 529$$

$$T_1 = 57.49 \text{ °C}$$

Heat flux to the room,

$$q''_{room} = q''_{cond} = \frac{57.49 - 27}{2.22} = 13.74 \ W / m^2 \qquad \text{...(15.29)}$$

T_6:

$$\frac{T_1 - T_6}{R_{cond}} = q''_{cond} = 13.74 \qquad \text{...(15.30)}$$

$$T_6 = 28.77 \text{ °C}$$

 # Single-pane glass window with external shading (based on SHGC calculation):

SHGC = 0.16 (for Suntex® 95 black shading, glass thickness ¼ inch = 6.35 mm)

Total resistance,

$$R_{total} = R_{conv,in} + \left(R_{cond} + R_{conv,out}\right) = \frac{1}{25} + 1.75 = 1.79 \frac{m^2K}{W} \qquad \text{...(15.31)}$$

Overall heat transfer coefficient,

$$u = \frac{1}{R_{total}} = 0.56 \frac{W}{m^2K} \qquad \text{...(15.32)}$$

Heat flux to the room,

$$q''_{room} = u\left(T_{\infty,o} - T_{\infty,i}\right) + SHGC \times q''_{solar} \qquad \text{...(15.33)}$$

$$q''_{room} = 0.56\left(29 - 27\right) + 0.16 \times 529 = 91.36 \ W / m^2$$

PROBLEM 16

The position of an occupant inside a room or office space affects his/her thermal comfort. Consider a multi-story office building located at Raipur. Two persons are sitting at different positions in one of the rooms/workspaces (dimension 10 × 5 × 4 m) in the building at 4.00 p.m. in the summer month of May. The surface area of each of the person is 1.6 m^2. The workspace is located in the intermediate floor of the building in such a way that only one of the walls (west wall) of the workspace is exposed to the hot outdoor environment. The exposed west wall (dimension 5 × 4 m) in this case is fully glazed with single-pane glass (8-mm thickness and SHGC ~0.4). Person 1 is sitting in the workspace near the glazed surface (west wall) and Person 2 is sitting near the north wall as shown in the figure. The shape factor associated with the two persons with different walls/surfaces of the building envelope are given in the following table. F_{XY} is the shape factor between X and Y, where X and Y represent person and wall/surface of the workspace envelope, respectively.

Table 16.1

Shape factors	Person 1	Person 2
F_{PN}	0.10	0.30
F_{PE}	0.05	0.06
F_{PS}	0.08	0.10
F_{PW}	0.27	0.06
F_{PC}	0.15	0.11
F_{PF}	0.35	0.37

The inner temperatures of the north wall, east wall, south wall, glazed west surface, ceiling, and floor surface are 31.1 °C, 31.0 °C, 31.1°C, 52.4 °C, 32.0 °C, and 31.8 °C, respectively. The indoor room air is maintained at 27 °C to achieve adequate thermal comfort. The average convective heat transfer coefficient associated with each person is 5 W/m^2K and the emissivity of the skin is 0.95. The mean skin temperature of both the persons and sweating rate can be taken as 34 °C and 50 g/h. The latent heat of evaporation of the sweat is 2500 kJ/kg. Neglecting the effect of clothing and respiration heat transfer, determine:

- Mean radiant temperature (MRT) and the operative temperature for Person 1 and Person 2.
- Net heat transfer (including convection, radiation, and evaporation) from each person considering steady state condition. Compare and discuss the results obtained for both persons.

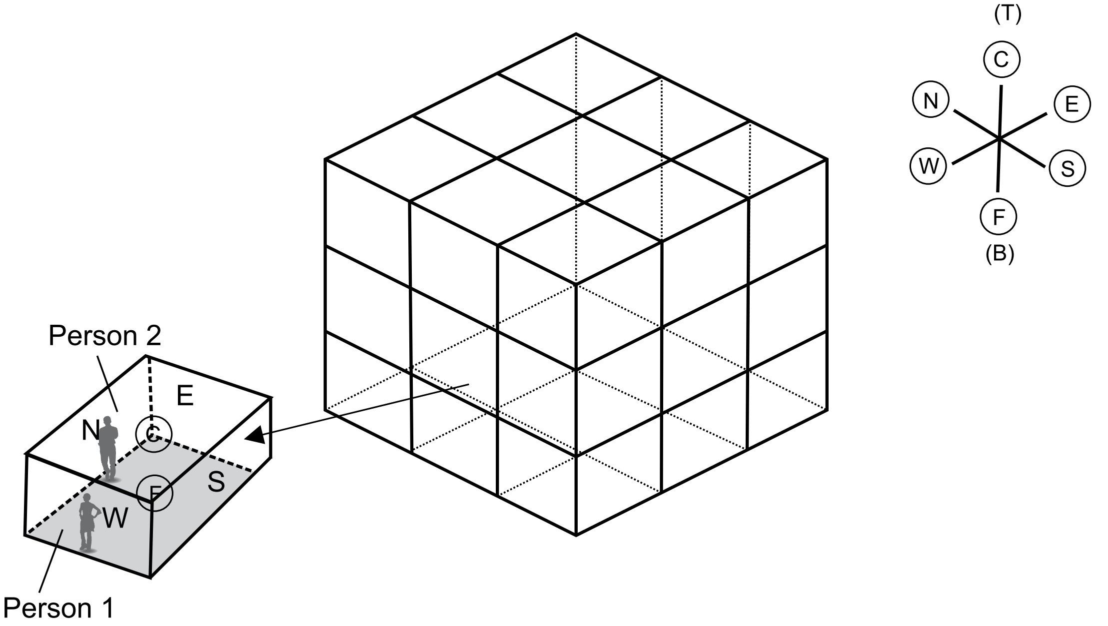

Figure 16.1

SOLUTION

Inner surface temperature of the north wall, T_N = 31.1 °C

Inner surface temperature of the east wall, T_E = 31.0 °C

Inner surface temperature of the south wall, T_S = 31.1 °C

Inner surface temperature of the west wall, T_W = 52.4 °C

Temperature of the ceiling, T_C = 32 °C

Temperature of the floor, T_F = 31.8 °C

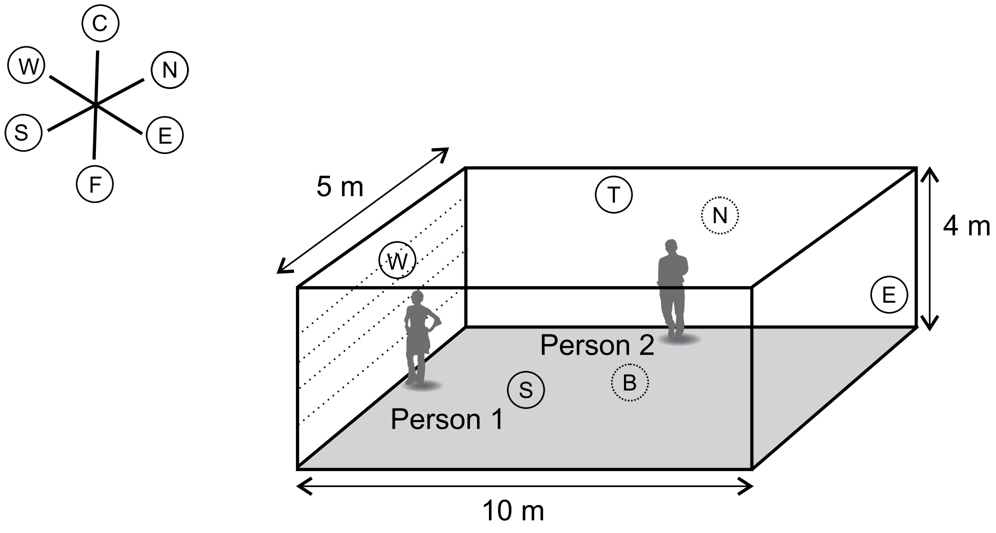

Figure 16.2

Shape factor for Person 1:

$F_{PN} = 0.10$, $F_{PE} = 0.05$, $F_{PS} = 0.08$, $F_{PW} = 0.27$, $F_{PC} = 0.15$, and $F_{PF} = 0.35$

Shape factor for Person 2:

$F_{PN} = 0.30$, $F_{PE} = 0.06$, $F_{PS} = 0.10$, $F_{PW} = 0.06$, $F_{PC} = 0.11$, and $F_{PF} = 0.37$

Heat transfer coefficient of the person, $h = 5$ W/m^{2}K

Emissivity of the person, $\varepsilon_P = 0.95$

Sweating rate, $\dot{m} = 50$ g/h

Latent heat of evaporation, $h_{fg} = 2500$ kJ/kg

Surface area of the person, $A_P = 1.6$ m^2

Indoor air temperature, $T_\infty = 27$ °C

Human skin temperature, $T_P = 34$ °C

(i) Person 1:

MRT:

$$MRT^4 = F_{PN}T_N^4 + F_{PE}T_E^4 + F_{PS}T_S^4 + F_{PW}T_W^4 + F_{PC}T_C^4 + F_{PF}T_F^4 \qquad \textbf{...(16.1)}$$

$$MRT^4 = 0.10(304.1)^4 + 0.05(304)^4 + 0.08(304.1)^4 + 0.27(325.4)^4 + 0.15(305)^4 + 0.35(304.8)^4$$

$$MRT = 310.65K = 37.65\ °\text{C}$$

Operative temperature,

$$T_{op} = \frac{MRT + T_\infty}{2} = 32.32\ °\text{C} \qquad \textbf{...(16.2)}$$

Radiation heat transfer,

$$q''_{rad} = 5.67\times10^{-8}\times0.95\left(307^4 - 310.65^4\right) = -23.16\ W/m^2 \qquad \textbf{...(16.3)}$$

Convective heat transfer,

$$q''_{conv} = 5(34-27) = 35\ W/m^2 \qquad \textbf{...(16.4)}$$

Evaporative heat transfer,

$$q_{eva} = \dot{m}h_{fg} = \frac{50}{1000 \times 3600} \times 2500 \times 10^{3} = 34.72\ W \qquad \text{...(16.5)}$$

$$q''_{eva} = \frac{q_{eva}}{A_P} = 21.7\ W/m^{2}$$

Net heat transfer from Person 1 to the room envelop,

$$q''_{net} = q''_{conv} + q''_{rad} + q''_{eva} = 35 - 23.16 + 21.7 = 33.54\ W/m^{2} \qquad \text{...(16.6)}$$

(ii) Person 2:

MRT:

$$MRT^{4} = F_{PN}T_{N}^{4} + F_{PE}T_{E}^{4} + F_{PS}T_{S}^{4} + F_{PW}T_{W}^{4} + F_{PC}T_{C}^{4} + F_{PF}T_{F}^{4} \qquad \text{...(16.7)}$$

$$MRT = 305.86K = 32.86\ ^{\circ}\mathrm{C}$$

Operative temperature,

$$T_{op} = \frac{MRT + T_{\infty}}{2} = 29.93\ ^{\circ}\mathrm{C} \qquad \text{...(16.8)}$$

Convective and evaporative heat transfers remain the same as in Person 1.

Radiation heat transfer,

$$q''_{rad} = 5.67 \times 10^{-8} \times 0.95\left(307^{4} - 305.86^{4}\right) = 7.07\ W/m^{2} \qquad \text{...(16.9)}$$

Net heat transfer from Person 2 to the room envelop,

$$q''_{net} = q''_{conv} + q''_{rad} + q''_{eva} = 35 + 7.07 + 21.7 = 63.77\ W/m^{2} \qquad \text{...(16.10)}$$

Therefore, the Person 2 will transfer heat around two times (63.77/33.54 = 1.9) more than Person 1, hence feel more thermally comfortable.

REMARKS

It can be observed that the two persons sitting in a same room can feel very different level of thermal comfort because of different locations in the room. Person 1 will feel less thermal comfort as he receives more radiation from the hot-glazed surface compared to Person 2 who is far away from the glazed surface. Heat released from Person 2 is around two times (63.77/33.54 = 1.9) higher compared to the heat released by Person 1. Hence, Person 2 will feel more thermal comfort.

PROBLEM 17

Radiant cooling technique is an energy-efficient option to achieve thermal comfort in buildings. Consider radiant cooling at the ceiling of a room located in a multi-storey building in Jaipur. Width, breadth, and height of the room are 5 m, 5 m, and 3.5 m, respectively. The cooled ceiling absorb heat from the room environment thus keep the room comfortable. Inner temperatures of various walls, ceiling, and floor were measured at 4.00 p.m. in the month of May. Temperatures of the ceiling and floor of the room are found to be 26 °C and 27 °C, respectively. The temperature of the west, east, north, and south walls are 32 °C, 29 °C, 29 °C and 29 °C, respectively. The emissivity of the floor is 0.9 and that of all other wall/roof surfaces is 0.8.

- Find out the net radiation heat transfer from the roof to the floor and from the roof to the east wall considering steady state condition.
- Repeat the calculations for a normal building construction with conventional air-conditioning system. Inner surface temperatures of the ceiling, floor, west wall, east wall, north wall, and south wall in this case are measured as 38 °C, 31 °C, 35 °C, 31 °C, 31 °C, and 31 °C, respectively.

SOLUTION

(i) Building with radiant cooling:

Given:

Temperature of the ceiling, $T_C = 26$ °C

Temperature of the floor, $T_F = 27$ °C

Temperature of the east wall, $T_E = 29$ °C

Temperature of the west wall, $T_W = 32$ °C

Temperature of the north wall, $T_N = 29$ °C

Temperature of the south wall, $T_S = 29$ °C

Emissivity of the floor, $\varepsilon_F = 0.9$

Emissivity of the other walls/roof, $\varepsilon = 0.8$

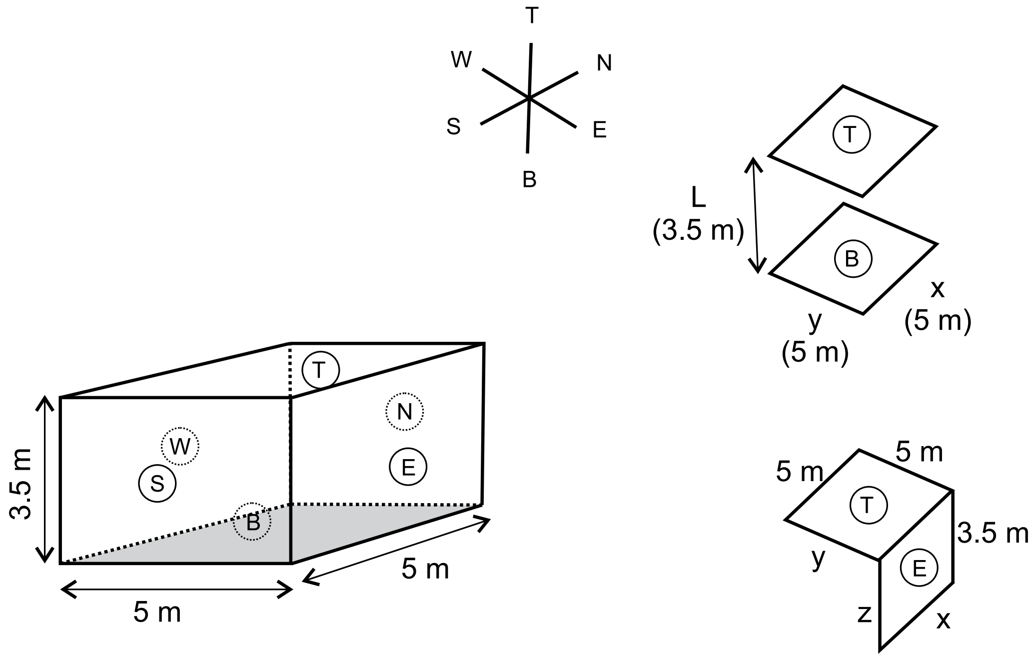

Figure 17.1

To find the shape factor or view factor (F_{ij}), X/L and Y/L need to be calculated. We can either use the formula (complex) or we can use the nomographs to find the shape or view factor, which can be found in standard heat transfer textbooks. For simplification, the values are given here directly.

For X/L = 1.43 and Y/L = 1.43, the shape factor from the ceiling to the floor is, $F_{CF} = 0.32$.

For Y/X = 1 and Z/X = 0.7, the shape factor from the ceiling to the east wall is, $F_{CE} = 0.17$.

Net radiation heat transfer from the ceiling to the floor:

Here, $A_T = A_B$

$$q_{CF} = \frac{\sigma\left(T_C^4 - T_F^4\right)}{\frac{1-\varepsilon_C}{A_C\varepsilon_C} + \frac{1}{A_C F_{CF}} + \frac{1-\varepsilon_F}{A_F\varepsilon_F}} \qquad \text{...(17.1)}$$

$$= \frac{5.67\times10^{-8}\left(299^4 - 300^4\right)\times 25}{\frac{1-0.8}{0.8} + \frac{1}{0.32} + \frac{1-0.9}{0.9}}$$

$$= -43.75\ W$$

Net radiation heat transfer from the ceiling to the east wall:

$$q_{CE} = \frac{\sigma\left(T_C^4 - T_E^4\right)}{\dfrac{1-\varepsilon_C}{A_C\varepsilon_C} + \dfrac{1}{A_C F_{CE}} + \dfrac{1-\varepsilon_E}{A_E\varepsilon_E}} \qquad \text{...(17.2)}$$

$$= \frac{5.67\times10^{-8}\left(299^4 - 302^4\right)}{\dfrac{1-0.8}{25\times0.8} + \dfrac{1}{25\times0.17} + \dfrac{1-0.8}{17.5\times0.8}}$$

$$= -69.85\ W$$

(ii) Normal building (without radiant cooling):

Given:

Temperature of the ceiling, T_C = 38 °C

Temperature of the floor, T_F = 31 °C

Temperature of the east wall, T_E = 31 °C

Net radiation heat transfer from the ceiling to the floor:

Here, $A_T = A_B$

$$q_{CF} = \frac{\sigma\left(T_C^4 - T_F^4\right)}{\dfrac{1-\varepsilon_C}{A_C\varepsilon_C} + \dfrac{1}{A_C F_{CF}} + \dfrac{1-\varepsilon_F}{A_F\varepsilon_F}} \qquad \text{...(17.3)}$$

$$= \frac{5.67\times10^{-8}\left(311^4 - 304^4\right)\times 25}{\dfrac{1-0.8}{0.8} + \dfrac{1}{0.32} + \dfrac{1-0.9}{0.9}}$$

$$= 331.66\ W$$

Net radiation heat transfer from the ceiling to the east wall:

$$q_{CE}=\frac{\sigma\left(T_C^4-T_E^4\right)}{\frac{1-\varepsilon_C}{A_C\varepsilon_C}+\frac{1}{A_CF_{CE}}+\frac{1-\varepsilon_E}{A_E\varepsilon_E}} \quad \textbf{...(17.4)}$$

$$=\frac{5.67\times10^{-8}\left(311^4-304^4\right)}{\frac{1-0.8}{25\times0.8}+\frac{1}{25\times0.17}+\frac{1-0.8}{17.5\times0.8}}$$

$$=174.88\ W$$

REMARKS

For radiant cooling case, radiant heat transfer is happening towards the ceiling, hence it receives heat from the floor, east wall, and other walls. But for conventional cooling case (a normal building construction), radiant heat transfer is occurring from the ceiling, thus it heats up the floor and walls.

PROBLEM 18

Consider a room in a building located in Raipur with exposed roof, west and south walls to the ambient environment. Width, breadth, and height of the room are 5 m, 5 m, and 3.5 m, respectively. A nude person having body surface area 1.8 m^2 stands at the centre of the room. The shape factor of the human body with all the side walls is 0.16. The shape factors of the human body with the floor and the ceiling are 0.24 and 0.12, respectively. Consider skin temperature, heat transfer coefficient, and emissivity of the human body as 34 °C, 8 W/m^2K, and 0.95, respectively. Emissivity of the floor is 0.9 and emissivity of all the other wall/roof surfaces is 0.8. Find out the total heat transfer (radiation + convection) from the human body to the building envelop for following two cases and discuss the results:

- Room has conventional air-conditioning system and indoor air temperature is maintained at 27 °C. Inner surface temperatures of ceiling, floor, east, west, north, and south walls are 45 °C, 30 °C, 30 °C, 32 °C, 30 °C, and 32 °C, respectively.

- Ceiling of the room has radiant cooling system and there is also a cooling/mechanical ventilation arrangement in the room to maintain indoor air temperature at 30 °C. Inner surface temperatures of ceiling, floor, east, west, north, and south walls are 26 °C, 27 °C, 29 °C, 30 °C, 29 °C, and 30 °C, respectively.

SOLUTION

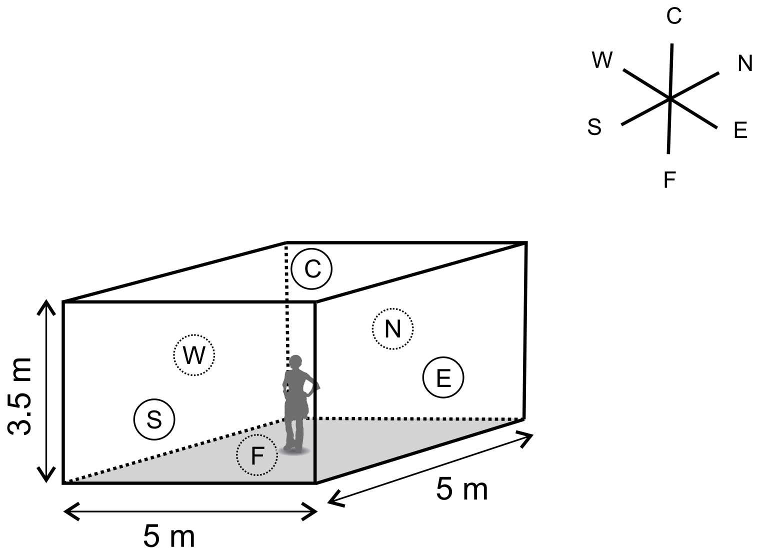

Figure 18.1

Given:

Dimension of the room = 5 × 5 × 3.5 m^3

Surface area of the person, $A_P = 1.8\ m^2$

Area of roof/floor, $A_C = A_F = 25$

Area of the walls, $A_E = A_W = A_N = A_S = 17.5$

Shape factor: (Person to room envelope)

$F_{PE} = F_{PW} = F_{PN} = F_{PS} = 0.16$, $F_{PF} = 0.24$, $F_{PC} = 0.12$

Temperature of the human body, $T_P = 34$ °C

Heat transfer coefficient of the person, $h = 8$ W/m^{2}K

Emissivity of the human skin surface, $\varepsilon_P = 0.95$

Emissivity of the floor surface, $\varepsilon_F = 0.9$

Emissivity of the other walls/roof, $\varepsilon = 0.8$

(i) Air-conditioned room:

Indoor air temperature, $T_\infty = 27$ °C

Temperature of the ceiling, $T_C = 45$ °C

Temperature of the floor, $T_F = 30$ °C

Temperature of the east wall, $T_E = 30$ °C

Temperature of the west wall, $T_W = 32$ °C

Temperature of the north wall, $T_N = 30$ °C

Temperature of the south wall, $T_S = 32$ °C

Net radiation heat transfer from man to ceiling:

$$q_{PC} = \frac{\sigma\left(T_P^4 - T_C^4\right)}{\dfrac{1-\varepsilon_P}{A_P\varepsilon_P} + \dfrac{1}{A_P F_{PC}} + \dfrac{1-\varepsilon_C}{A_C\varepsilon_C}} \qquad \textbf{...(18.1)}$$

$$= \frac{\sigma\left(T_P^4 - T_C^4\right)A_P}{\dfrac{1-\varepsilon_P}{\varepsilon_P} + \dfrac{1}{F_{PC}} + \dfrac{1-\varepsilon_C}{\varepsilon_C}\left(\dfrac{A_P}{A_C}\right)}$$

$$= -\frac{5.67\times10^{-8}\left(318^4 - 307^4\right)1.8}{\dfrac{1-0.95}{0.95} + \dfrac{1}{0.12} + \dfrac{1-0.8}{0.8}\left(\dfrac{1.8}{25}\right)}$$

$$= -16.32\ W$$

Net radiation heat transfer from man to floor:

$$q_{PF} = \frac{5.67\times10^{-8}\left(307^4 - 303^4\right)1.8}{\frac{1-0.95}{0.95}+\frac{1}{0.24}+\frac{1-0.9}{0.9}\left(\frac{1.8}{25}\right)} \quad \text{...(18.2)}$$

$$=10.95\ W$$

Net radiation heat transfer from man to east/north wall:

$$q_{PE} = q_{PN} = \frac{5.67\times10^{-8}\left(307^4 - 303^4\right)1.8}{\frac{1-0.95}{0.95}+\frac{1}{0.16}+\frac{1-0.8}{0.8}\left(\frac{1.8}{17.5}\right)} \quad \text{...(18.3)}$$

$$=7.33\ W$$

Net radiation heat transfer from man to west/south wall:

$$q_{PW} = q_{PS} = \frac{5.67\times10^{-8}\left(307^4 - 305^4\right)1.8}{\frac{1-0.95}{0.95}+\frac{1}{0.16}+\frac{1-0.8}{0.8}\left(\frac{1.8}{17.5}\right)} \quad \text{...(18.4)}$$

$$=3.70\ W$$

Net radiation heat transfer from man to walls, ceiling, and floor,

$$q_{net,P} = -16.32+10.95+2\times7.33+2\times3.70 \quad \text{...(18.5)}$$

$$=16.69\ W$$

Convective heat transfer from the person,

$$q_{conv} = 8\left(34-27\right)1.8 = 100.8\ W \quad \text{...(18.6)}$$

Total heat transfer from the person,

$$q_{total} = q_{rad} + q_{conv} = 16.69+100.8 = 117.49\ W \quad \text{...(18.7)}$$

Heat flux,

$$q'' = \frac{q_{total}}{A_P} = \frac{117.49}{1.8} = 65.27W/m^2 \quad \text{...(18.8)}$$

(ii) Room with radiant cooling:

Indoor air temperature, T_∞ = 30 °C

Temperature of the ceiling, T_C = 26 °C

Temperature of the floor, T_F = 27 °C

Temperature of the east wall, T_E = 29 °C

Temperature of the west wall, T_W = 30 °C

Temperature of the north wall, T_N = 29 °C

Temperature of the south wall, T_S = 30 °C

Net radiation heat transfer from the person to the ceiling:

$$q_{PC} = \frac{\sigma\left(T_P^4 - T_C^4\right)A_P}{\frac{1-\varepsilon_P}{\varepsilon_P} + \frac{1}{F_{PC}} + \frac{1-\varepsilon_C}{\varepsilon_C}\left(\frac{A_P}{A_C}\right)} \quad \textbf{...(18.9)}$$

$$= \frac{5.67\times10^{-8}\left(307^4 - 299^4\right)1.8}{\frac{1-0.95}{0.95} + \frac{1}{0.12} + \frac{1-0.8}{0.8}\left(\frac{1.8}{25}\right)}$$

$$= 10.82\ W$$

Net radiation heat transfer from the person to the floor:

$$q_{PF} = \frac{5.67\times10^{-8}\left(307^4 - 300^4\right)1.8}{\frac{1-0.95}{0.95} + \frac{1}{0.24} + \frac{1-0.9}{0.9}\left(\frac{1.8}{25}\right)} \quad \textbf{...(18.10)}$$

$$= 18.89W$$

Net radiation heat transfer from the person to the east/north wall:

$$q_{PE} = q_{PN} = \frac{5.67\times10^{-8}\left(307^4 - 302^4\right)1.8}{\frac{1-0.95}{0.95} + \frac{1}{0.16} + \frac{1-0.8}{0.8}\left(\frac{1.8}{17.5}\right)} \quad \textbf{...(18.11)}$$

$$= 9.12\ W$$

Net radiation heat transfer from the person to the west/south wall:

$$q_{PW} = q_{PS} = \frac{5.67\times10^{-8}\left(307^4 - 303^4\right)1.8}{\frac{1-0.95}{0.95} + \frac{1}{0.16} + \frac{1-0.8}{0.8}\left(\frac{1.8}{17.5}\right)} \quad \textbf{...(18.12)}$$

$$= 7.33\ W$$

Net radiation heat transfer from the person to walls, ceiling, and floor,

$$q_{net,P} = 10.82 + 18.89 + 2 \times 9.12 + 2 \times 7.33 \quad \text{...(18.13)}$$

$$= 62.61\ W$$

Convective heat transfer from the person,

$$q_{conv} = 8(34 - 30)1.8 = 57.6\ W \quad \text{...(18.14)}$$

Total heat transfer from the person,

$$q_{total} = q_{rad} + q_{conv} = 62.61 + 57.6 = 120.21\ W \quad \text{...(18.15)}$$

Heat flux,

$$q'' = \frac{q_{total}}{A_P} = \frac{120.21}{1.8} = 66.78\ W / m^2 \quad \text{...(18.16)}$$

REMARKS

It can be observed that in the case of conventional air-conditioning system, the total heat flux from the person is 65.27 W/m^2, which is almost the same (66.78 W/m^2) as that of the radiant cooling system. Hence, the radiant cooling system and the conventional system are providing similar levels of thermal comfort. However, the indoor air temperature is 30 °C in the radiant cooling case and it is 27 °C in the conventional air-conditioning system. Reducing the indoor air temperature by just 1 °C reduces the energy consumption up to 10% (Hoyt *et al.*, 2015; doi: https://doi.org/10.1016/j.buildenv.2014.09.010). The results shown here shows that the air need not be cooled much in the case of radiant cooling system. Higher indoor air temperature requirement for the radiant cooling system indicates lower energy requirement in the case of radiant cooling system as compared to the conventional air-conditioning system. Thus, it can be concluded that radiant cooling technique is a better and efficient technique for achieving thermal comfort.

In the conventional air-conditioning system, the entire cooling is done by one system, which operates at a relatively lower temperature and hence the efficiency of cooling system is lower. Also, the amount of air circulation is higher, leading to higher fan energy consumption. On the other hand, a major part (60%–80%) of the cooling requirement is handled by radiant heat transfer to the cooling system placed in the ceiling in the case of radiant cooling system. The remaining part (20%–40%) of the cooling is provided by the conventional cooling system, where lesser amount of air needs to be circulated, leading to reduced fan and overall energy consumption. Hence, the conventional air-conditioning system can operate at higher temperature when a radiant cooling system is installed, thus consuming lower energy.

PROBLEM 19

Consider a man standing at the centre of a room of a two-story building, located in Jaipur, India. Width, breadth, and height of the room are 5 m, 5 m, and 3.5 m, respectively. Roof, west and south walls of the room are exposed to the outdoor ambient environment at 1.00 p.m. in the month of May. The inner surface temperatures of east, west, north, and south walls are 30 °C, 32 °C, 30 °C, and 32 °C, respectively. The surface area of the human body is 1.8 m^2 and the shape factor of the human body with all the side walls is 0.16. The shape factor of the human body with the floor and the ceiling are 0.24 and 0.12, respectively. Calculate the net radiation heat transfer from the human body considering that:

- Person is standing in a room 'A' located in the upper floor of the building, and
- Person is standing in another room 'B' located in the ground floor of the building.

Room 'B' is located just below room 'A'. The ceiling and floor temperatures of room 'A' are 45 °C and 30 °C, respectively. The ceiling and floor temperatures of room 'B' are 30 °C and 29 °C, respectively. Consider temperature and emissivity of the human body as 34 °C and 0.95, respectively. The emissivity of the floor is 0.9 and the emissivity of all the other wall/roof surfaces is 0.8.

SOLUTION

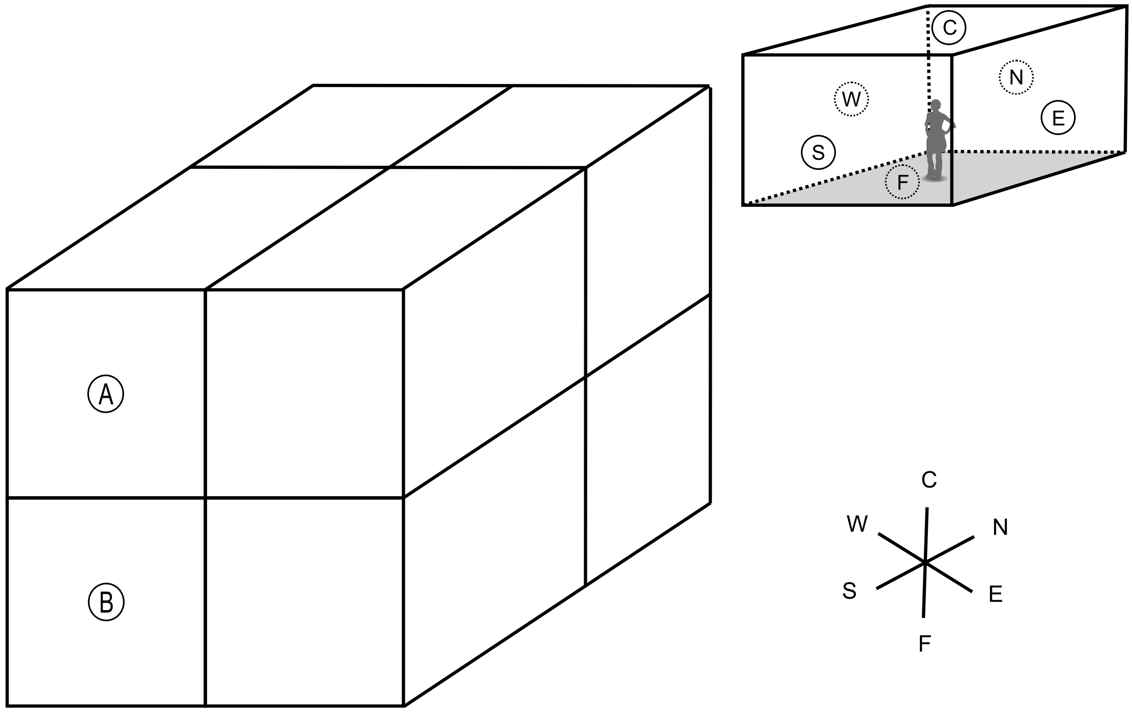

Figure 19.1

(i) Upper floor (Room A):

Given:

Temperature of the ceiling, T_C = 45 °C

Temperature of the floor, T_F = 30 °C

Temperature of the east wall, T_E = 30 °C

Temperature of the west wall, $T_W = 32$ °C

Temperature of the north wall, $T_N = 30$ °C

Temperature of the south wall, $T_S = 32$ °C

Temperature of the human body, $T_M = 34$ °C

Shape factor:

$F_{ME} = F_{MW} = F_{MN} = F_{MS} = 0.16$, $F_{MF} = 0.24$, $F_{MC} = 0.12$

Emissivity of the human body, $\varepsilon_M = 0.9$

Emissivity of the floor, $\varepsilon_F = 0.9$

Emissivity of the other walls/roof, $\varepsilon = 0.8$

Surface area of the human body, $A_M = 1.8\ \text{m}^2$

Area of roof/floor, $A_C = 25\ \text{m}^2$

Area of the walls, $A_E = A_W = A_N = 1.75\ \text{m}^2$

Net radiation heat transfer from the person to the ceiling:

$$q_{MC} = \frac{\sigma\left(T_M^4 - T_C^4\right)}{\frac{1-\varepsilon_M}{A_M\varepsilon_M} + \frac{1}{A_M F_{MC}} + \frac{1-\varepsilon_C}{A_C\varepsilon_C}} \qquad \textbf{...(19.1)}$$

$$= \frac{\sigma\left(T_M^4 - T_C^4\right)A_M}{\frac{1-\varepsilon_M}{\varepsilon_M} + \frac{1}{F_{MC}} + \frac{1-\varepsilon_C}{\varepsilon_C}\left(\frac{A_M}{A_C}\right)}$$

$$= -\frac{5.67\times10^{-8}\left(318^4 - 307^4\right)1.8}{\frac{1-0.95}{0.95} + \frac{1}{0.12} + \frac{1-0.8}{0.8}\left(\frac{1.8}{25}\right)}$$

$$= -16.32\ W$$

Net radiation heat transfer from the person to the floor:

$$= \frac{5.67\times10^{-8}\left(307^4 - 303^4\right)1.8}{\frac{1-0.95}{0.95} + \frac{1}{0.24} + \frac{1-0.9}{0.9}\left(\frac{1.8}{25}\right)} \qquad \textbf{...(19.2)}$$

$$= 10.95\ W$$

Net radiation heat transfer from the person to the east/north wall:

$$=\frac{5.67\times10^{-8}\left(307^4-303^4\right)1.8}{\frac{1-0.95}{0.95}+\frac{1}{0.16}+\frac{1-0.8}{0.8}\left(\frac{1.8}{17.5}\right)} \qquad \text{...(19.3)}$$

$$=7.33\ W$$

Net radiation heat transfer from the person to the west/south wall:

$$=\frac{5.67\times10^{-8}\left(307^4-305^4\right)1.8}{\frac{1-0.95}{0.95}+\frac{1}{0.16}+\frac{1-0.8}{0.8}\left(\frac{1.8}{17.5}\right)} \qquad \text{...(19.4)}$$

$$=3.70\ W$$

Net radiation heat transfer from the man to walls, ceiling, and floor,

$$q_{net,M}=-16.32+10.95+2\times7.33+2\times3.70 \qquad \text{...(19.5)}$$

$$=16.69\ W$$

(ii) Ground floor (Room B):

Temperature of the ceiling, T_C = 30 °C

Temperature of the floor, T_F = 29 °C

All other temperatures and parameters remain unchanged.

Net radiation heat transfer from the person to the ceiling:

$$=\frac{5.67\times10^{-8}\left(307^4-303^4\right)1.8}{\frac{1-0.95}{0.95}+\frac{1}{0.12}+\frac{1-0.8}{0.8}\left(\frac{1.8}{25}\right)} \qquad \text{...(19.6)}$$

$$=5.52\ W$$

Net radiation heat transfer from the person to the floor:

$$=\frac{5.67\times10^{-8}\left(307^4-302^4\right)1.8}{\frac{1-0.95}{0.95}+\frac{1}{0.24}+\frac{1-0.9}{0.9}\left(\frac{1.8}{25}\right)} \qquad \text{...(19.7)}$$

$$=13.62\ W$$

Net radiation heat transfer from the person to the east/north wall and the west/south wall remains the same as of the upper floor, i.e. 7.33 W and 3.70 W.

Net radiation heat transfer from the person to walls, ceiling, and floor,

$$q_{net,M} = 5.52 + 13.62 + 2 \times 7.33 + 2 \times 3.70 \quad \textbf{...(19.8)}$$

$$= 41.2 \ W$$

The increase in heat transfer (radiation) from the person to the room envelop in the ground floor is around 2.5 times (41.2/16.69 = 2.47) as compared to the top floor.

REMARKS

For the upper floor, the net radiation heat transfer from the person to the building envelop is 16.69 W (or 9.27 W/m^2). In the ground floor, the net radiation heat transfer from the human body to the building envelop is 41.2 W (or 22.89 W/m^2). Human normal metabolic heat generation in indoor conditions for various activities like resting, walking, and other office activities is in the range of 40 to 120 W/m^2. To maintain human heat balance and thermal comfort, the heat generated inside the human body is to be released. Various modes of heat transfer to release the heat from the body are radiation, convection, conduction, evaporation, and respiration. The results obtained in the present problem show that the radiation heat transfer on the ground floor is ~2.5 times that of the upper floor. So, in order to maintain thermal comfort in the upper floor (Room A), all or some of the other modes of heat transfer (evaporation – by sweating and convection – by fans) need to be increased in the upper floor as compared to the ground floor (Room B). It results in the higher energy consumption in terms of either mechanical ventilation or HVAC system. In the absence of any mechanical ventilation or HVAC system, the person on the top floor will sweat more while the person on the ground floor will feel more comfortable.

PROBLEM 20

Room environment and building envelop affect human thermal comfort. Consider a room with all side walls and roof exposed to the hot environment. Consider two configurations of the exposed wall/roof: configuration 1 and configuration 2. In configuration 1, all the exposed walls and roof are uninsulated and correspond to the normal building construction. In configuration 2, all the exposed walls and roof incorporate thermal insulation to reduce the U-value or heat transfer through envelop.

- Mean radiant temperature (MRT), which is defined as a uniform temperature of an imaginary enclosure (building envelop) is found to be 35 °C and 29 °C for configuration 1 and configuration 2, respectively. A person (height 1.7 m and surface area 1.8 m^2) is standing in a relaxed posture (metabolic heat rate 70 W/m^2) in a room maintained at 27 °C. Human body can be considered as a cylindrical arrangement with the core and three surrounding layers, namely epidermis, dermis, and subcutaneous (fat) layers. It may be assumed that metabolic heat generation takes place only in the central core region. Average convective heat transfer coefficient associated with a nude human body is 8 W/m^2K and the emissivity of the skin is 0.95. Considering a sweating rate of 50 g/h, determine the body core and exposed skin surface temperatures corresponding to both the wall configurations. Also, calculate the convective, radiative, and evaporative heat flux to/from the human body. Compare and discuss the results.

Thickness of the epidermis, dermis, and subcutaneous layers of the skin are 0.08 mm, 2.0 mm, and 10 mm, respectively. Thermal conductivity of the epidermis, dermis, and subcutaneous layers of the skin are 0.24 W/mK, 0.45 W/mK, and 0.19 W/mK, respectively. Latent heat of evaporation of the sweat is 2500 kJ/kg. For simplification:

- Human body is assumed to cylindrical. As the top and bottom areas of the cylindrical human body are very small compared to the overall surface area, the heat transfer from top and bottom portions of the cylindrical human body is neglected.
- The core region of the body remains at a uniform temperature.
- The sweating rate remains constant irrespective of the body core temperature.
- Heat transfer through conduction to the floor is neglected.
- Surface area of the human body can be assumed to be very small compared to the enclosure area. So, it can be assumed as a case of radiation exchange between a small object kept in a large enclosure.

SOLUTION

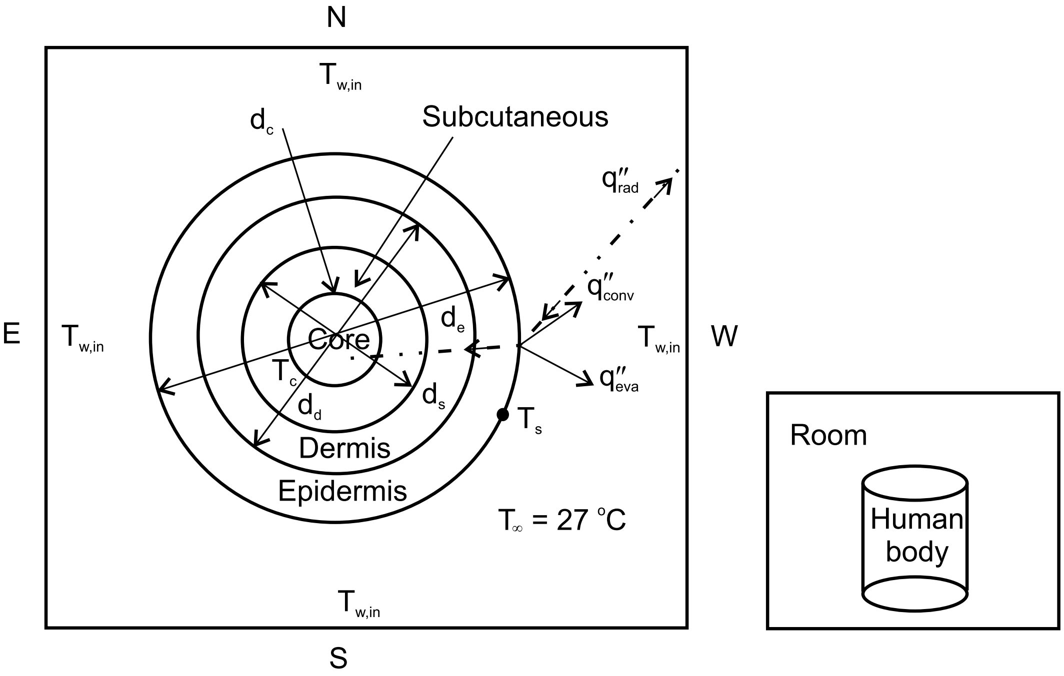

Figure 20.1

Given:

Thermal conductivity and thickness of the epidermis layer, $k_e = 0.24$ W/mK, $L_e = 8 \times 10^{-5}$ m

Thermal conductivity and thickness of the dermis layer, $k_d = 0.45$ W/mK, $L_d = 0.002$ m

Thermal conductivity and thickness of the subcutaneous layer, $k_s = 0.19$ W/mK, $L_s = 0.01$ m

Metabolic heat rate, $q''_{mh} = 70$ W/m^2

Height of the cylindrical human body, $L = 1.7$ m

Surface area of the human body, $A = 1.8$ m^2

Diameter of the cylindrical human body, d: (equal to outer diameter of the epidermis layer)

$$d = d_e = \frac{1.8}{\pi \times 1.7} = 0.337\ m$$

Outer diameter of the dermis layer, $d_d = 0.3368$ m

Outer diameter of the subcutaneous layer, ds = 0.3328 m

Diameter of the core, $d_c = 0.3128$ m

Heat transfer coefficient of the nude human body, $h = 8$ W/m^{2}K

Emissivity of the skin, $\varepsilon = 0.95$

Sweating rate, $\dot{m} = 50$ g/h

Latent heat of evaporation, h_{fg} = 2500 kJ/kg

Evaporative heat transfer, q_{eva}

$$q_{eva} = q''_{eva} \times A = \dot{m} \times h_{fg} = 34.72\ W$$

Indoor room temperature, T_∞ = 27 °C

Total thermal resistance,

$$R_{cond} = \frac{\ln\frac{d_e}{d_d}}{2\pi k_e L} + \frac{\ln\frac{d_d}{d_s}}{2\pi k_d L} + \frac{\ln\frac{d_s}{d_c}}{2\pi k_s L}$$

$$R_{cond} = \frac{\ln\frac{0.337}{0.3368}}{2\pi \times 0.24 \times 1.7} + \frac{\ln\frac{0.3368}{0.3328}}{2\pi \times 0.45 \times 1.7} + \frac{\ln\frac{0.3328}{0.3128}}{2\pi \times 0.19 \times 1.7}$$

$$R_{cond} = 2.32 \times 10^{-4} + 2.48 \times 10^{-3} + 0.03 = 0.0332 K/W$$

From the heat balance at the outer skin for steady state condition:

$$q_{eva} + q_{conv} + q_{rad} = q_{cond} = q''_{mh} \times A \qquad \textbf{...(20.1)}$$

(i) Case I: MRT = 35 °C

Let the body core and exposed skin surface temperatures be T_c and T_s, respectively.

$$q''_{mh} \times A = q_{cond} \qquad \textbf{...(20.2)}$$

$$70 \times 1.8 = \frac{T_c - T_s}{R_{cond}} = \frac{T_c - T_s}{0.0332}$$

$$T_c = T_s + 4.18$$

$$q''_{mh} \times A = q_{eva} + q_{conv} + q_{rad} \qquad \textbf{...(20.3)}$$

$$70 \times 1.8 = 34.72 + 8 \times 1.8\left(T_s - 300\right) + 5.67 \times 10^{-8} \times 0.95 \times 1.8\left(T_s^4 - 308^4\right)$$

$$9.7 \times 10^{-8} \times T_s^4 + 14.4 \times T_s + 34.72 - 4320 - 872.53 - 126 = 0$$

$$9.7 \times 10^{-8} \times T_s^4 + 14.4 \times T_s - 5283.81 = 0$$

$$T_s = 307.05K = 34.05\ °C$$

Therefore,

$$T_c = T_s + 4.18 = 38.23 \text{ °C} \qquad \textbf{...(20.4)}$$

Evaporative heat flux from the human body, q''_{eva}

$$q''_{eva} = 34.72 / 1.8 = 19.29 \ W/m^2 \qquad \textbf{...(20.5)}$$

Convective heat flux from the human body, q''_{conv}

$$q''_{conv} = 8(307.05 - 300) = 56.4 \ W/m^2 \qquad \textbf{...(20.6)}$$

Radiative heat flux from the human body, q''_{rad}

$$q''_{rad} = 70 - 19.29 - 56.4 = -5.69 \ W/m^2 \qquad \textbf{...(20.7)}$$

Here, negative sign signifies heat transfer to the human body.

(i) Case II: MRT = 29 °C

In this case also, from equation 20.1, we can get following two equations as obtained in the above case.

$$q''_{mh} \times A = q_{cond} \qquad \textbf{...(20.8)}$$

$$T_c = T_s + 4.18$$

And,

$$q''_{mh} \times A = q_{eva} + q_{conv} + q_{rad} \qquad \textbf{...(20.9)}$$

$$70 \times 1.8 = 34.72 + 8 \times 1.8 (T_s - 300) + 5.67 \times 10^{-8} \times 0.95 \times 1.8 \left(T_s^4 - 302^4\right)$$

$$9.7 \times 10^{-8} \times T_s^4 + 14.4 \times T_s - 5218.14 = 0$$

$$T_s = 304.48K = 31.48 \text{ °C}$$

Therefore,

$$T_c = T_s + 4.18 = 35.66 \text{ °C} \qquad \textbf{...(20.10)}$$

Evaporative heat flux from the human body, q''_{eva}

$$q''_{eva} = 34.72 / 1.8 = 19.29 \ W/m^2 \qquad \textbf{...(20.11)}$$

Convective heat flux from the human body, q''_{conv}

$$q''_{conv} = 8(31.48 - 27) = 35.84 \ W/m^2 \qquad \textbf{...(20.12)}$$

Radiative heat flux from the human body, q''_{rad}

$$q''_{rad} = 70 - 19.29 - 35.84 = 14.87\ W/m^2 \qquad \textbf{...(20.13)}$$

REMARKS

Based on relatively simple calculations, it can be observed that for uninsulated walls, radiative heat transfer occurs from the indoor wall to the skin. Due to higher inner wall surface temperature and associated radiation heat transfer to the human body, the person is feeling relative hot, which is evident from the higher body core temperature. Body core temperature in this case was 38.23 °C against the normal temperature range, 36.5–37.5 °C. In the case of insulated wall assembly, radiation heat transfer occurs from the skin to the wall, as skin temperature was higher than the inner wall surface temperature. Due to additional heat release from the human body caused by radiation, skin and core temperatures were found to be lower than the previous case. It is also observed that the convective heat transfer is dominating in both the cases with nude human body compared to radiative and evaporative heat transfer.

About the Book

This book introduces undergraduate/postgraduate (UG/PG) engineering students to the building science area. To make concepts clearer to students, 20 problems and their solutions have been presented in this book. These problems are developed with a view that they serve as good examples for the professional core subject, Heat and Mass Transfer, taught in engineering. Students' understanding of the fundamental concepts of this core subject will improve further when they apply themselves to solve these problems. The problems presented include examples from various topics of the subject such as conduction, convection, and radiation modes of heat transfer while touching some of the basic aspects related to thermal comfort and passive building design strategies. Necessary care has been taken, and appropriate assumptions have been made at various places in a few problems to simplify them, so that UG/PG students can understand and appreciate them while learning the core ideas of building physics. This book would make concepts clearer even to practising architects and engineers.

About Bureau of Energy Efficiency

Bureau of Energy Efficiency (BEE) is a statutory body under the Ministry of Power, Government of India. It assists in developing policies and strategies with the primary objective of reducing the energy intensity of the Indian economy. BEE coordinates with designated consumers, designated agencies, and other organizations to identify and utilise the existing resources and infrastructure in performing the functions assigned to it under the Energy Conservation Act.

About the Indo-Swiss Building Energy Efficiency Project

The Indo-Swiss Building Energy Efficiency Project (BEEP) is a bilateral cooperation project between the Ministry of Power, Government of India, and the Federal Department of Foreign Affairs of the Swiss Confederation. The overall goal of the project is to reduce energy consumption in new commercial, public, and residential buildings in India through energy-efficient and thermally comfortable design. The project has four key components: building design, building technologies, building policy, and outreach.

For further information

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